Socially Optimal Crime and Punishment

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Abstract

This paper develops a dynamic life-cycle equilibrium model of crime with heterogeneous agents and human capital accumulation. Agents decide at each point in time whether to commit crimes by comparing potential gains from crime to the expected loss due to the probability of apprehension and the associated cost (incarceration and reduced future wages in the legal labor market). Public security policies are defined as pairs of a size of the police force and an average length of sentences. We propose an original micro-founded public security technology linking the level of police expenditures to the probability of arrest. Plugging this technology in the dynamic equilibrium framework allows us to evaluate public security policies. Equilibrium effects can be potentially relevant because of dynamic interactions between the classical incapacitation and deterrence effects. Using this model, we explore the optimality of policies in a way that would not be possible with reduced form empirical estimates or with the traditional, partial equilibrium, static, theoretical models of crime. We conduct an exploratory quantitative exercise calibrating the model to US property crime data from the 2000s. The calibrated model points to overspending in police protection and overincarceration in that period, when compared to the optimal public security policy.

1 Introduction

Crime imposes large costs on societies. In the US, expenditures on the criminal justice system in 2012 added up to at least 210 billion dollars and the cost of crime to victims has been estimated to be roughly of this same order of magnitude.[1] At the same time, an increasing amount of evidence suggests that crime is very responsive to policy, be it related to the presence and distribution of police personnel, to the duration of sentences, or to social welfare programs.[2] In fact, the design of policies to fight crime has historically relied heavily

[1] Using victimization data from the FBI and NCVS combined to estimations of the average cost of each type of crime taken from Cohen (2000), we estimate an aggregate cost of victimization in the US of the order of 240 billion. This number varies greatly depending on the methodology, but, if anything, 240 billion is close to a lower bound.

[2] For examples of papers measuring the impact of policing on the crime rate, see e.g. Marvell and Moody (1996), Levitt (1997), McCrary (2002), Levitt (2002), Di Tella and Schargrodsky (2004), Klick and Tabarrok

on empirical evidence and it seems fair to say that, in a certain way, it is almost a model for evidence-based policy making.

Virtually all of the evidence that has informed the design of crime policies, however, is based on randomized-control trials and reduced form estimates, with few, if any, equilibrium considerations. Most of the times, relevant dynamic aspects are also ignored and the analysis is conducted in what, for all practical purposes, is a static setting. But crime is a phenomenon that depends heavily on equilibrium responses, so it is likely that different policies interact in non-trivial ways. At the same time, crime-fighting policies may have dynamic implications that are themselves relevant in determining the optimal policy design. Longer sentences, for example, reduce the number of potential criminals on the streets in the short run, likely increasing the effectiveness of a given police force, which in turn affects the optimal size and allocation of police. But longer sentences also reduce the future employability of inmates, increasing their likelihood of future recidivism. Are these trade-offs relevant from a policy perspective? If so, which one does dominate in the long run?

Cost-benefit analyses based on results from reduced-form estimates cannot hope to answer questions such as these. So, despite the vast array of evidence on the effectiveness of various crime interventions, we know little about the global optimality of crime policies and, most importantly, do not even share a canonical theoretical framework with which to address these types of questions. Some of these trade-offs have certainly been conceptually identified, in one way or another, since the classic theoretical contributions to the economics of crime (for example, (Becker 1968) and (Ehrlich 1981)). But, since then, not much has been done to bring these dimensions together in a manageable equilibrium model able to help inform policy and to shed light on the relevance of the trade-offs across these various margins.

This paper contributes to the literature by proposing an equilibrium framework to evaluate public safety policies. Public safety policies in our context are understood as referring to the size of the police force and the length of sentences. We consider a dynamic equilibrium life-cycle model with heterogeneous agents and human capital accumulation where agents decide at each point in time whether to commit crimes. This decision is based on the comparison between the potential gain from crime and the expected loss due to a positive probability of being caught and the associated punishment (duration of incarceration). A crime is modelled as having the following consequences: (i) a transfer of resources from victims to criminals (the crime itself); (ii) a non-monetary utility loss to victims (the psychological cost of being victim of a crime); and (iii) a net loss of resources (the potential destruction or use of resources during the event of a crime). To match the data and to allow for a non-economic dimension of heterogeneity, we assume that there is a mass of individuals who contemplate committing crimes and a mass of individuals who, irrespective of the circumstances, never commit crimes (maybe due to moral considerations).

The model leads to a simple intuitive characterization of the decision to commit crimes:

(2005), Draca et al. (2008). For examples of the impact of incarceration on the level of crime via incapacitation, see for example Marvell and Moody (1994), Levitt (1996), Kessler and Levitt (1999), Buonanno and Raphael (2013), Owens (2009), Barbarino and Mastrobuoni (2014). For papers studying the effect of welfare transfers in crime rates, see Zhang (1997), Hannon and DeFranzo (1998), Foley (2008), Jacob and Ludwig (2010) and Chioda et al. (2016).

at each point in time there is a cut-off level of income below which individuals who consider committing crimes indeed engage in criminal activity, conditional on a given set of public security policies and on the decisions of other agents. Since we also incorporate life-cycle considerations, this decision changes as individuals age and their incomes and horizon change. We characterize a rational expectations equilibrium in this economy and show that it exists. Once calibrated using data and evidence from reduced-form empirical studies, our model allows us to simulate the impact of any policy alternative and, in particular, to determine the socially optimal public safety policy and crime level.

A key challenge in addressing this problem, and maybe one of the reasons why quantitative theoretical work on the economics of crime has lagged behind, is how to model the "public security production function" available to policy makers. In other words, how do expenditures on police translate into a probability of apprehending criminals?[3] The idea of making the apprehension probability endogenous to public security expenditures was first introduced by Ehrlich (1973). Ehrlich (1973) proposes a public security production function in which the probability of punishment is a function of expenditures on police and the crime rate. The functional form he proposes is appealing at the local level, when considering small deviations from a given initial point, but for larger deviations can lead to probabilities that exceed 1. İmrohoroğlu et al. (2000) adopts a similar power functional that, instead, can lead to negative probabilities and deals with this problem simply by truncating the apprehension probability whenever the expression is smaller than 0. To our knowledge, these are the only papers in the literature that explicitly model the public security production function. The limitations of the approach adopted in both these papers lie on the arbitrariness of both the functional form and, from a quantitative perspective, the way the parameters are calibrated.

Our first contribution to the literature is to propose a simple micro-founded public security production function. Our technology is motivated by a model in which criminals choose the location of crimes and policymakers choose where to deploy police forces in order to maximize the number of apprehensions. In addition, we assume that each apprehension is time costly for the police units. Under this assumption, we show that, given the size of the police force, the probability of apprehension of any individual criminal is decreasing in the total number of criminals (under excessive load, police units are less effective to apprehend criminals). In our setting, this can be interpreted as the game played between criminals and police, conditional on the individual decision to commit crimes and on the size of the police force. The probability of apprehension arising from the equilibrium in this game is a two-parameter closed-form function depending on the size of the police (or, alternatively, on expenditures on police) and on the crime rate. This function respects all expected properties, including those discussed by Ehrlich (1973) and İmrohoroğlu et al. (2000). In addition, it has the advantage of allowing us to calibrate the relevant parameters directly from the estimates of the elasticity of crime with respect to prison population and police expenditures

[3]In the last two decades, according to the Bureau of Justice Statistics, approximately 45% of the expenditures of the American justice system were devoted to police forces, 20% allocated to the judicial system and other legal activities, and 35% allocated to correctional institutions. Therefore the relevance of what we call the public security production function and the need to deal with it explicitly in the context of our theory.

available from the empirical literature (such as Levitt (1996) and Levitt (2002)). Our quantitative exercise, therefore, is informed by the evidence from reduced-form empirical studies. In this sense, it also helps bridge the large gap that currently exists between the theoretical and empirical literatures in economics of crime.

This technology also has the appealing feature of naturally bringing into the picture social interactions and the possibility of multiple equilibria in crime rates. Since the probability of apprehension of any given criminal is a direct function of the number of active criminals, the possibility of having social interactions as an important dimension of the crime phenomenon is present in the model in a completely micro-founded way. The potential relevance of social interactions in the determination of equilibrium levels of crime has been highlighted before both theoretically and empirically (see, for example, Gleaser et al. (1996), Burdett et al. (2003), and Gaviria (2000), each one with a different rationalization for the presence of social interactions).[4]

Another key feature of our dynamic model is the incorporation of a life-cycle component. The explicit incorporation of a realistic earnings profile over the life-cycle is essential for the model to be able to reproduce the widely documented age profile of participation into crime (see, for example, discussion in Beirne (1993), Lochner (2004), and Munyo (2013)). It is also important to incorporate the changes in the cost of crime that are driven by reductions in future legal earnings due to incarceration. Grogger (1995), for example, suggests that one year of incarceration can reduce wages by as much as 30% in the long run. This labor market channel amplifies the deterrent power of incarceration in non-trivial ways and makes it highly age dependent. We summarize life-cycle productivity changes with a learning-by-doing human capital accumulation technology, somewhat similar to Flinn (1986). We assume that individuals accumulate skills as a by-product of work and whenever they are in prison their productivity is depreciated. Since we abstract from educational policies, the interaction of safety policies and criminal behavior leads to implications similar to those that would be obtained in a more standard Becker-Ben Porath investment model of human capital.

The combination of these ingredients introduces relevant equilibrium interactions that, otherwise, would have been impossible to consider. For example, in principle it is possible that increasing the length of sentences leads to an increase in the steady-state crime rate, due to the positive effect of incarceration on recidivism through reduced future earnings. Whether or not this mechanism is relevant depends on the current equilibrium in the economy and on interactions among various policy dimensions, which are impossible to assess without an explicit life-cycle equilibrium model.

We calibrate our model using US property crime data in 2004. As mentioned before, we also use estimates from reduced-form empirical studies to calibrate the parameters of our public security production function. Since we are considering rational criminals motivated by the economic return to crime, property crime seems like the natural focus. We choose to calibrate our model in 2004 since this year lies in the middle of a period ranging from 2000 to 2010 in which the crime trends were relatively stable, being therefore more compatible with the stationarity hypothesis we used to solve our model. For a wide range of policy

[4]In our context, we prove the existence of at least one equilibrium for any set of parameters.

alternatives and starting points considered, our calibration procedure leads to a single point in the parameter space and the equilibrium seems to be unique. The optimal safety policy characterized by the model would imply a higher crime rate, fewer individuals incarcerated, and a yearly monetized social welfare gain of almost 4 billion dollars. Maybe surprisingly, this would be achieved by spending considerably less in police protection and punishing property crimes slightly more harshly (in other words, punishing with slightly harsher sentences considerably fewer individuals). The reduction of expenditures on police combined with lower aggregate costs of incarceration results in a much cheaper public security system, which translates into a non-trivial welfare gain for tax-payers.

At the same time, we obtain two very counterintuitive conclusions. First, although the apprehension probability is much lower and there are fewer prisoners, in aggregate criminals experience a reduction in welfare under the suggested policy. This happens because the lower apprehension probability of the new policy increases the net expected gain in criminal activities, encouraging criminals with higher productivity—to whom incarceration imposes a higher social loss—to commit crimes. Second, the optimal safety policy implies an increase in the crime rate, which goes against the natural assumption that the final purpose of a public security policy is always to reduce crime.

We also conduct quantitative exercises that acknowledge that policy makers may be politically constrained in their set of instruments or objectives. Our structural model allows us to consider any such policy and also to determine the optimal social policy under different welfare functions. So, for example, we also discuss optimal public security policies that keep the initial crime rate constant or that do not include the criminal's benefit from committing crimes in the social welfare function. Reassuringly, for all exercises considered, we find that the optimal policy would reduce expenditures on police and increase slightly the severity of punishment, which, in every case, would end up reducing the steady-state prison population. Even if each exercise—whether changing the constraint or the social welfare metric—solves a different problem, the qualitative consistency of our results suggests some degree of robustness in our main quantitative findings.

Finally, our quantitative model also sheds new light on the classical decomposition of the effects of public security policies. Besides illustrating theoretically the role of the incapacitation, deterrence, and rehabilitation effects, the model highlights that, when we consider equilibrium responses, there is an additional effect that has not yet been considered in the literature. We call it the *load effect*. The *load effect* refers to the lower effectiveness of a given police force under higher crime rates, due to the dilution of human and physical resources when trying to catch a larger number of criminals. By lower effectiveness we mean the lower probability of apprehension of any individual criminal when the total number of criminals is higher, given a constant amount of resources allocated to the police. Notice that this is, by nature, an equilibrium effect, therefore requiring an equilibrium framework even to be conceptually defined. We formally define these four effects in the context of our model and calculate the role of each in determining the effect of public security policies on crime.

As mentioned before, the conceptual question of optimal law enforcement is by no means new. The first normative consideration of law enforcement can be traced back to Hobbes in Leviathan, published in 1651. Hobbes argued that the rational and self-interested nature of people imply that the punishment for crime must be greater than the benefit that comes from committing this crime. Beccaria went further in *Dei Delitti e delle penne* and concluded that there should be a universal scale of punishments as a function of the crime committed—acknowledging the challenge of finding such scale. Bentham posited that the duty of the state was "to promoting the happiness of the society, by punishing and rewarding". Indeed, the seminal paper on the economics of crime of Becker (1968) deals precisely with this normative problem and Ehrlich (1981) also addresses this question. Differently from our work, the models proposed by these authors—as nearly all theoretical models of crime until 2000—are static and do not attempt to explicitly model the public security production function. So, despite being useful to illustrate the nature of some of the issues that arise when considering the question of an optimal public security policy, their use is limited in actual policy analysis.[5]

The papers closest to our work are Engelhardt et al. (2008) and Fella and Gallipoli (2014). Both these papers develop dynamic equilibrium models of crime, simulate some public safety policies, and make normative considerations. Engelhardt et al. (2008) focus on the relationship between the labor market and criminal behavior. Two among the several exercises they proposed are closely related to our paper, one being the impact of alternative sentence lengths in the economy and the other being the impact of alternative apprehension probabilities. Because they do not model the life-cycle, their model is not able to capture rehabilitation, the recidivism effect of imprisonment, or the age-profile of criminal involvement. Moreover, they use the same arbitrary apprehension technology proposed by İmrohoroğlu et al. (2000), therefore missing the load effect and, even more important, being restricted to have acceptable simulations only for small changes in the public safety policy. Fella and Gallipoli (2014) analyze the relationship between crime and education. They do incorporate life-cycle considerations, but take the probability of apprehension of criminals as exogenous. In addition, since their main focus is on education, in their model incarceration does not imply any loss of productivity. For these reasons, we believe that the use of their model to analyze the optimality of public security policies is limited.

Needless to say, our model has many limitations. First, we consider only one type of crime and do not allow for substitution across alternative types of crime (robberies, thefts, burglaries, bank robberies, etc.). Though allowing for this possibility would not represent a major challenge from a theoretical perspective (see, for example, Ferraz (2017)), it is already difficult to calibrate our model considering only one type of crime, so it would be a major challenge at this stage to expand the quantitative exercise to consider multiple types of crime. Second, we do not incorporate non-economic crimes or behavioral considerations in the decision to commit crimes. Our choice to focus on property crime has the objective of starting from what seems to be the most appealing case for economic analysis. And

[5]Several, but not all, characteristics of dynamic models can be captured by static ones. For example, for any time of incarceration, there is a fine that would cause the same deterrence effect. But the incapacitation effect of incarceration for an arbitrary safety policy can only be generated by a dynamic equilibrium model. In addition, the dynamic interaction between current punishment and future recidivism, through reduced future labor market earnings, also requires a dynamic model. Finally, issues related to the life-cycle profile of criminal involvement can obviously not be dealt with in the context of a static model.

third, we do not explicitly model the justice system and the sentencing process, having a direct link between the apprehension of a criminal and the associated punishment (without any possibility of wrongful convictions, for example). Though all of these dimensions are potentially important, we see our contribution as a first step towards building a manageable and useful dynamic model of crime, which may help bring together empirical evidence and equilibrium considerations when thinking about the optimality of public security policies. The points raised in this paragraph are natural extensions to follow in the future of this research agenda.

The remainder of the paper is organized as follows. Section 2 presents the model and defines an equilibrium. Section 3 describes the calibration of the parameters using as benchmark property crimes in the US economy in 2004. Section 4 presents the main quantitative exercises and discusses the optimal public safety policy in the context of the model. Section 5 concludes the paper.

2 The model

We consider a dynamic equilibrium model with heterogeneous agents where crime is the primary source of inefficiency. Time is discrete and we consider only the stationary state. Below, we describe the economy of our framework.

2.1 Preferences

The economy is populated by a continuum of individuals who are ex ante heterogeneous with respect to their productivity. Individuals are risk neutral. At each period, each individual maximizes the expected discounted lifetime utility

$$\mathbb{E}\Big[\sum_{t=t'}^{\infty}\Big(\prod_{\tau=t'}^{t}(\beta(1-\delta_\tau))c_t^i\Big)\Big],$$

where β denotes the intertemporal discount factor, δ_t is the probability of death at age t and c_t^i denotes the consumption of individual i of age t. In this economy, individuals are able to commit crimes. However, committing crime, for many agents and for a number of reasons (cultural, psychological or ethical ones), is highly costly and only a fraction of the agents actually consider this possibility. We call these types honest and dishonest agents. At each period, a constant number of new individuals of each type at age 0 enter this economy. The productivity of each agent at age 0 is a positive number drawn from a distribution depending only on the type of the agent. The number of dishonest individuals at age 0 entering this economy equals M_0 with cumulative distribution of productivities given by G_0, continuous, with support $(0, \infty)$ and probability density function denoted by g_0. Honest agents are passive, they matter for the social welfare and are the only targets of criminal actions in this economy. Since, in addition, all agents are risk neutral, the (utilitarian) social loss criminal actions inflict upon those individuals depends only on the crime rate. Hence, the social welfare is independent of the number of honest agents in this economy as well as of their income distribution.

2.2 Legal Sector and Crime

Let $s_t^i \in \{\tilde{E}, E, P\}$ denote the three possible states for a dishonest agent i at age t. If $s_t^i = \tilde{E}$, the agent is in freedom and not engaged in criminal activities. If $s_t^i = E$, the agent is in freedom and engaged in criminal activities. If $s_t^i = P$, the agent is in prison. The only choice dishonest agents make is whether or not to engage in crime. This choice is made at the beginning of each period, whenever they start a period in freedom. Honest agents make no decisions.

There is only one type of crime in this economy. This crime is an interaction between a dishonest agent at some age t (the criminal) and a honest agent (the victim), resulting in three consequences: a financial transfer value z from the victim to the criminal representing the market value of the stolen property, a damage d^V inflicted on the victim representing physical or psychological damages and an age-dependent cost d_t^C paid by the criminal. This latter cost represents the psychological pain for committing a crime, the material investment on the crime or the opportunity cost for abstaining working in the legal sector when they are committing crimes.

If a dishonest individual i chooses to engage in crime at age t, this individual remains engaged in crime until the end of the period. In this case, an intra-period continuous-time stochastic process takes place. The agent has successive opportunities of crime. The interval between two opportunities within a period is a random variable exponentially distributed with parameter ν (on units of the discrete period), independent of everything else. The individual i who decides to be a criminal commits a crime in every opportunity until the occurence of the first of two events: the current period ends or he is apprehended.

The apprehension of a criminal for a given crime can only occur at the instant (of the intra-period) this crime was committed. Every crime is subjected to detection[6] with a probability p. The exponentially distributed interval between two opportunities implies that, if agent i at age t is not apprehended for any crime, the number of crimes J_t^i in this period is a random variable distributed as Poisson with parameter ν. Being apprehended for some crime is the logical negation of not being apprehended for any crime, which means that, conditional on J_t^i, the probability of apprehension at the period is $1-(1-p)^{J_t^i}$. Hence, for J_t^i distributed as Poisson, the expected probability of being apprehended in a period is given by

$$\mathbb{E}\left[1-(1-p)^{J_t^i}\right] = \sum_{j=0}^{\infty}\left((1-(1-p)^j)\frac{\nu^j e^{-\nu}}{j!}\right) = 1 - e^{-\nu p}.$$

An agent i at age t engaged in crimes consumes an extra $z - d_t^C$ for every crime he successfully commits in the current period. In case of detection, there is no financial transfer

[6] We make a distinction between detection and apprehension. The former is an action on a crime, the latter is an action on an agent and all the subsequent procedures taken by the police after the detection of a crime.

between the victim and the criminal, but the victim still suffers the damage d^V and the criminal still pays the cost d_t^C. Given these assumptions, the gain from crimes in the current period is a random variable depending on the age of the agent denoted as Π_t^i. We can use the Poisson distribution of J_t^i to calculate the expected gains from crime at the beginning of a period. For a given J_t^i, if the agent is apprehended for the j-th (in chronological order) crime, $1 \leq j \leq J_t^i$, his gain is $(j-1)z - jd_t^C$. The apprehension probability at the j-th crime is $(1-p)^{j-1}p$, so

$$\mathbb{E}\left[\Pi_t^i | J_t^i\right] = \frac{(z(1-p) - d_t^C)(1-(1-p)^{J_t^i})}{p}.$$

Thus, the expected unconditional gain from crimes in the current period is given by

$$\overline{\Pi}_t := \mathbb{E}[\Pi_t^i] = \frac{(z(1-p) - d_t^C)(1-e^{-\nu p})}{p}.$$

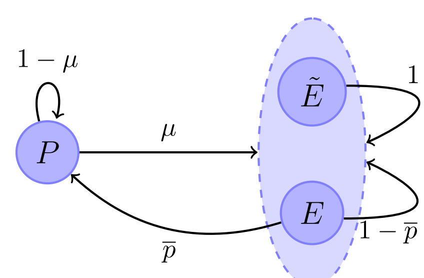
When an agent is apprehended or when an agent starts a period in prison, there is a probability $1-\mu$ that he will start the next period in prison, and a probability μ that he will start next period in freedom. We ignore the possibility of acquittals for apprehended individuals, so spending 0 periods in prison captures the case when convicted criminals are incarcerated for very short periods. One of the roles of the government in this economy is to set this probability μ, which is the same as setting the average sentence length $\frac{1}{\mu} - 1$.

In this economy, each agent in freedom earns an income[7] equal to his productivity. The only way engagement in crime affects income is through the cost d^C. We denote w_t^i the productivity of the dishonest agent i at age t. All individuals (even the ones in prison) pay a lump-sum tax f. So, the expected consumption of an agent i at age t is given by

$$\mathbb{E}\left[c_t^i\right] = \begin{cases} w_t^i - f & \text{if } s_t^i = \tilde{E}, \\ w_t^i - f + \overline{\Pi}_t & \text{if } s_t^i = E, \\ b - f & \text{if } s_t^i = P, \end{cases}$$

where b is the pre-tax level of consumption of a convicted criminal. Given these assumptions and defining $\overline{p} := (1-\mu)(1-e^{-\nu p})$, we can obtain the state transition probabilities conditional on the state of the agents:

$$\begin{aligned} \Pr(s_{t+1}^i \in \{\tilde{E}, E\} | s_t^i = \tilde{E}) &= 1 \\ \Pr(s_{t+1}^i \in \{\tilde{E}, E\} | s_t^i = E) &= 1 - \overline{p} \\ \Pr(s_{t+1}^i = P | s_t^i = E) &= \overline{p} \\ \Pr(s_{t+1}^i \in \{\tilde{E}, E\} | s_t^i = P) &= \mu \\ \Pr(s_{t+1}^i = P | s_t^i = P) &= 1 - \mu \end{aligned}$$



[7] We use the shorter name "income" to refer to "income obtained in the legal sector".

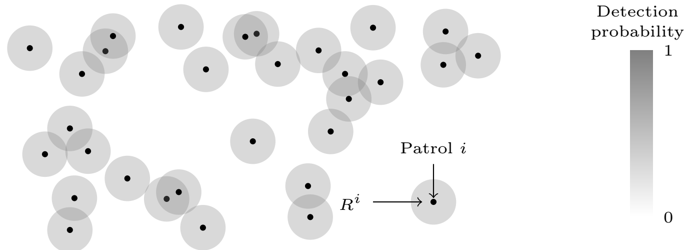


Figure 1: Apprehension probability as a function of the position for a given configuration of patrolling police units.

To capture life-cycle earning profiles, we assume that the productivity of an individual at age t increases by a factor $\gamma_t \geq 1$. For some agent T, agents at age $t \geq T$ have constant productivity, i.e., $\gamma_t = 1$. On the other hand, due to stigma and depreciation of human capital, the productivity of individuals decrease by a factor θ for each period they spend in prison. The dynamics of the productivity of agent i can be summarized as

$$w^i_{t+1} = \begin{cases} \gamma_t w^i_t & \text{if } s^i_t \in \{\tilde{E}, E\}, \\ \theta w^i_t & \text{if } s^i_t = P. \end{cases}$$

2.3 Apprehension Technology

The technology available to apprehend criminals is as follows. There is a region R where all crimes occur. To deter and detect crimes, police units are deployed in this region. A police unit is able to detect, with probability q, any crime committed within a circle, of radius r, centered at his position. Figure 1 illustrates the detection probability for a given configuration of police units. Upon detecting a crime, government incurs a cost of α to apprehend the criminal.

At any time, the number of police units is distributed as Poisson with parameter Λ. Conditional on the number of police units, each police units is deployed independently with a distribution D over R. The government can control two properties of this deployment. One, is the parameter of the Poisson distribution, which is proportional to police expenditures devoted to patrolling k^{patrol}, $\Lambda = \xi k^{\text{patrol}}$. In addition, government can control the probability density D. While the social planner seeks to choose D in order to maximize the proportion of crimes that are detected for a given k^{patrol}, each criminal chooses the location of his crime in order to minimize the probability of being detected.[8] The only Nash equilibrium this game allows is one where the social planner sets the distribution D to be uniform—otherwise all criminals would choose to commit crimes in the area where the density of patrols is the lowest—and criminals choose their crimes such that the density of crimes in the region is constant—otherwise the social planner would choose D higher in the regions with higher

[8]In equilibrium, the proportion of crimes that are detected and individual probability of being detected are equal, and we call them "apprehension probability".

incidence of crime. This equilibrium produces an apprehension probability (see Appendix A for more details) given by the following expression,

$$p(v,k) = 1 - \frac{W_L(\zeta_2 v e^{-\zeta_1 k + \zeta_2 v})}{\zeta_2 v}, \tag{1}$$

where W_L is the Lambert-W function, v is the crime rate, k are the police expenditures and ζ_1 and ζ_2 are positive constants dependent on q, α, ξ, r, and the area of R.[9]

We highlight two points regarding our apprehension model. First, by using a micro-founded apprehension technology, we can predict the state of the economy for a wide range of policy alternatives. Hitherto, all other dynamic models of crime have relied upon a single arbitrary functional form, $p^I(v,k) = \max\{0, 1-k^{-\Gamma}\}$, to relate expenditures on police protection to the apprehension probability. This functional form, proposed by İmrohoroğlu et al. (2000), is attractive since it is continuous, increasing in police expenditures, bounded in the $[0,1]$ interval, goes to 0 when the police expenditures is small and goes to 1 when police expenditures are high, all properties that we expect for the causal effect of the police expenditures on the apprehension probability. However, being an ad-hoc functional conceived to respect a set of properties, p^I can be rivalled by any other functional respecting the same properties—including functionals leading to different results. This concern is mitigated when counter-factual policies are close enough to the current one, but the validity of p^I cannot be evaluated otherwise. In opposition to İmrohoroğlu et al., the validity of our apprehension probability lies on the apprehension model hypotheses we have introduced.

Second, rather than using a set of expected properties to shape our functional, we use them to support the validation of our apprehension model. The function p we have obtained respects simultaneously, the properties proposed by İmrohoroğlu et al. (2000), another set of properties proposed by Ehrlich (1973), and an additional property that posits the number of apprehensions, $vp(v,k)$, is increasing in the crime rate.[10]

2.4 Incarceration Technology

The incarceration technology—the prisons—has a cost linear on the number of prisoners, so it costs κN^p at a period to keep a mass N^p of prisoners incarcerated at each period, where $\kappa > 0$. This implies that the total expenditures on the public security system at each period is given by $k + M^p\kappa$.

2.5 Definition of a Stationary Equilibrium

We consider the stationary rational expectation equilibrium. There are two state variables for an agent i at age t: the first one to indicate whether the agent is in prison or in freedom and the second one corresponding to his productivity. Besides, the control variable resumes

[9] We define $p(0,k) = \lim_{v\to 0} p(v,k) = 1 - e^{-\zeta_1 k}$.

[10] Ehrlich (1973) posits that a higher crime rate causes a lower apprehension probability with the apprehension probability going to 0 as the crime rate goes to infinity.

to the choice of whether to engage in criminal activities, subject to not engaging if the agent is in prison. Let $V_t^F(w)$ be the value function of an agent at age t, having productivity w, starting the period in freedom and $V_t^P(w)$ the analogous for an agent starting the period in prison. If $\tilde{\beta}_t := \beta(1-\delta_t)$, the dynamic programming problem faced by a dishonest agent with productivity w at age t can be written as

$$V_t^F(w) = w - f + \max\Big\{\tilde{\beta}_t V_{t+1}^F(\gamma_t w), \overline{\Pi}_t + \tilde{\beta}_t(1-\overline{p})V_{t+1}^F(\gamma_t w) + \tilde{\beta}_t \overline{p} V_{t+1}^P(\gamma_t w)\Big\}. \quad (2)$$

Regardless of the choice of engagement in crime, each agent in freedom earns his income (for legal activities), pays the lump-sum tax and has his productivity increased by a factor γ_t depending only on his age. Agents not engaged in crime at age t will be free at age $t+1$. Agents engaged in criminal activities have an expected gain from crimes given by Π_t and may not be caught and start next period in freedom, expressed by the term $(1-\overline{p})V_{t+1}^F(\gamma_t w)$, or may be caught and start the next period in prison, expressed by $\overline{p}V_{t+1}^P(\gamma_t w)$. The value function of an agent in prison is

$$V_t^P(w) = b - f + (1-\mu)\beta(1-\delta_t)V_{t+1}^P(\theta w) + \mu\beta(1-\delta_t)V_{t+1}^F(\theta w). \quad (3)$$

It is possible to obtain the characterization of the dishonest agents' behavior. From Equation (2), an agent with productivity w at age t strictly prefers to engage in crime if

$$\overline{\Pi}_t > \overline{p}(V_{t+1}^F(\gamma_t w) - V_{t+1}^P(\gamma_t w)).$$

The left-hand term trivially expresses the expected differential utility gain obtained from criminal activities, which is constant in the income. The losses of being convicted for a crime include the opportunity cost of exchanging his income for the consumption of an incarcerated criminal and the permanent reduction of productivity. These losses are expressed by the term $V_{t+1}^F(\gamma_t w) - V_{t+1}^P(\gamma_t w)$, so the right-hand term expresses the expected differential utility losses from engagement in crime. We notice that both losses are increasing in the productivity. Thus, for each age, the agent's solution for the programming problem depends only whether his income is above a age-dependent cut-off income w_t^* solving

$$\overline{\Pi}_t = \overline{p}(V_{t+1}^F(\gamma_t w_t^*) - V_{t+1}^P(\gamma_t w_t^*)).$$

An agent at age t strictly prefers to engage in crime if, and only if, his income is below w_t^*. Agents at age t with income above w_t^* do not engage in crime. More details and a formal proof of this behavior are given in Appendix B.

Definition 2.1. *A stationary equilibrium for a given choice of public security policy (μ, k) is a collection of value functions $V_t^P(w)$, $V_t^F(w)$, individual policy rules of engagement or non engagement in criminal activities by age and income, time-invariant measures of dishonest agents $m_t^s(w)$ for each income $w \in (0,\infty)$, each age $t \in \{0,1,\ldots\}$ and each state $s \in \{\tilde{E}, E, P\}$, an aggregate crime rate v^*, an apprehension probability $p^* := p(v,k)$, and a lump-sum tax f such that:*

(i) Individual and aggregate behavior are consistent, i.e., the aggregate crime rate is given by

$$\sum_{t=0}^{\infty}\int_0^{\infty} m_t^E(w)(1-p^*)(1-e^{-\nu p^*})/p^*\,dw.$$

(ii) Individual and aggregate state changes are consistent.

(iii) Given the public security policy (μ, k)*,* f *and a belief in* p*, the individual policy rules of engagement or non engagement in criminal activities solve the individuals' dynamic program defined by equations* (2) *and* (3)*.*

(iv) Expectations are rational, i.e., the common belief in the apprehension probability is p^**.*

(v) Government runs a balanced budget, so the tax collection via lump-sum f *equals the expenditures on the public security policy* $k + \kappa \sum_{t=0}^{\infty}\int_0^{\infty} m_t^P(w)\,dw$*.*

It is possible to show that there is always at least one stationary equilibrium in this economy. Appendix B presents the proof and also the idea behind the computation of the stationary equilibrium.

2.6 Welfare

Crime is the primary source of inefficiency in this economy, caused by the deadweight loss implied by each criminal action. The government reduces the social burden caused by crime through the public security technologies at its disposal.[11]

In the stationary state, the social planner considers the aggregate consumption of all agents in this economy $\int c^i(k,\mu)\mathrm{d}i$, where (k,μ) is the policy set by the social planner and $c^i(k,\mu)$ is the consumption of agent i under this policy. We define $\widetilde{W}$ as the counterfactual aggregate consumption if there were no crimes and no prisoners and frame the social planner's problem as a function of the social welfare loss

$$L(k,\mu) = \widetilde{W} - \int_{\substack{\text{All}\\ \text{agents}}} c^i(k,\mu)\mathrm{d}i.$$

The social planner seeks to minimize the loss of welfare caused by crime and public security policies. The total loss incurred in the public sector is the sum of the expenditures on the apprehension technology $L^{\text{pol}}(k,\mu) := k$, and the cost incarceration $L^{\text{keep}}(k,\mu) := \kappa N^P(k,\mu)$, where $N^P(k,\mu)$ is the total number of incarcerated agents. Due to condition *(v)* of equilibrium, the public sector loss is entirely financed by the individuals of this economy.

[11] An environment where agents are not risk averse would take in account the dimension of fear caused by the possibility of victimization.

The direct crime loss is given by

$$L^{\text{crime}}(k,\mu) := \sum_{t=0}^{\infty} v_t(k,\mu)(d^V + d_t^C),$$

where $v_t(k,\mu)$ is the equilibrium number of crimes committed by agents at age t under policy (k,μ). We also consider the loss inflicted on apprehended criminals from both freedom deprivation $L^{\text{FD}}(k,\mu)$ (i.e., from foregone incomes at all periods they are in prison), and depreciation of productivity $L^{\text{HK}}(k,\mu)$.

Combining all these losses, the social planner's problem is to choose the policy that solves

$$(k^{**},\mu^{**}) = \min_{(k,\mu)} \left\{ L^{\text{pol}}(k,\mu) + L^{\text{keep}}(k,\mu) + L^{\text{crime}}(k,\mu) + L^{\text{FD}}(k,\mu) + L^{\text{HK}}(k,\mu) \right\}. \quad (4)$$

The expressions for $L^{\text{FD}}(k,\mu)$ and $L^{\text{HK}}(k,\mu)$ are derived in the Appendix C.

There are some points regarding the aggregation of consumption $\int c^i(k,\mu)\mathrm{d}i$ that merit a discussion. First, because this economy is in the stationary state and is populated by a continuum of agents, at each period the aggregation of the consumption matches exactly $\int c^i(k,\mu)\mathrm{d}i$. Second, given a social intertemporal discount factor β_S, the (socially) discounted aggregate consumption over time is equal to $1/(1-\beta_S)\int c^i(k,\mu)\mathrm{d}i$, therefore maximizing welfare in one period is enough to maximize the discounted aggregate consumption for any β_S, including the intertemporal discount factor β. With linear utilities, $1/(1-\beta)\int c^i(k,\mu)\mathrm{d}i$ is both the discounted money-metric (or wealth) of this economy and also the utilitarian metric. However, because the model lacks a market structure, this money metric does not suffer from the reference dependence problem.[12]

3 Parametrization

We describe in this section the procedures to obtain all the parameters of the model. Some parameters can be directly obtained from other articles and official datasets and we deal with them in Section 3.1. Further details of this parametrization are left for Appendix D. Then, in section 3.2, we find the remaining parameters by matching some quantities describing the US economy in 2004 with the equivalents moments in our model. Section 3.3 evaluates the accuracy of the matching of moments and also compares some non-targeted data moments with the equivalent quantities in the model.

We are interested in crimes with economic motivations, so we consider as property crimes only robbery, burglary, motor theft and over 50 dollars larceny[13]. We exclude low value larcenies due to their low impact in the social welfare—less then 0.1% of the social loss caused by property crimes—and we exclude fraud since the technologies to commit or detect white collar crimes differ enough from the ones to commit or detect street crimes.

[12]See Blackorby and Donaldson (1988) and Donaldson (1992) for a deeper discussion about the reference problem.

[13]This is a different definition Bureau of Justice Statistics (BJS) and the Federal Bureau of Investigation (FBI) use. They consider robbery as a violent crime and include all types of larceny.

We benchmark our model to match parameters from external sources and to match moments of 2004 whenever data for this year is available. We choose a year after 2000 as a benchmark because the number of prisoners and the crime rate are relatively stable after this year, which is more consistent with the stationarity hypothesis imposed in the model. This stability is illustrated in Figure 2, taken from the National Prisoners Survey (NPS), and Figure 3, taken from the National Criminal Victimization Survey (NCVS). We choose the specific year of 2004 to the parametrization due to a higher availability of data in this year.

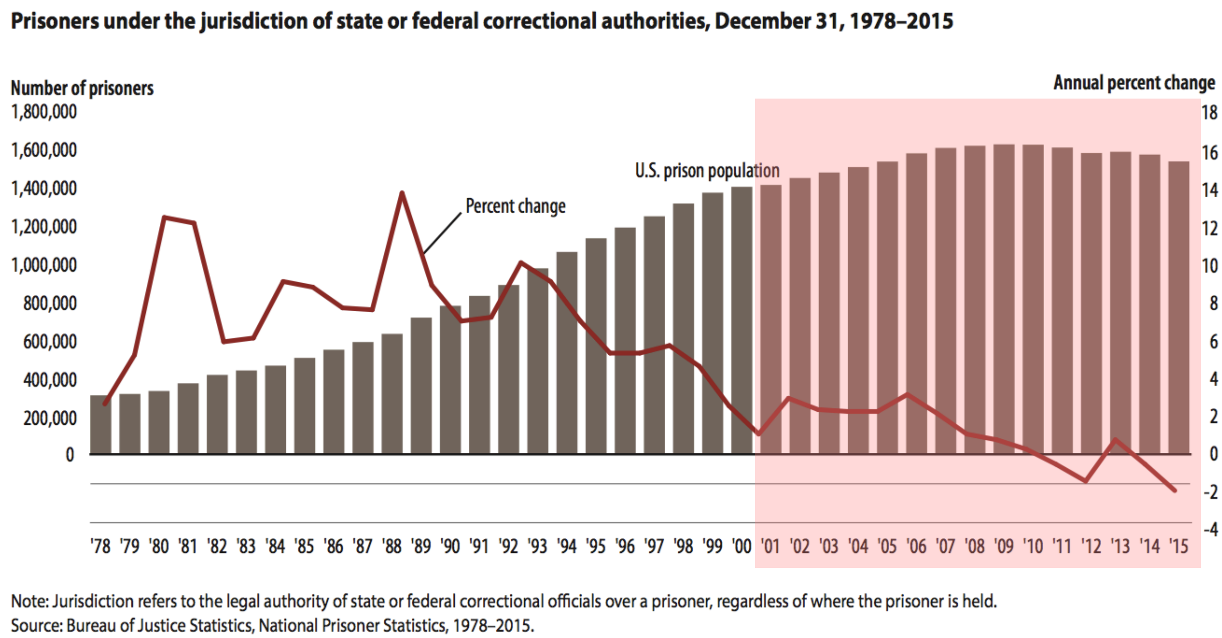


Figure 2: Number of prisoners is relatively steady after 2001

3.1 Parameters from external sources

The unit of time is one year. Since individuals under 18 years old are mostly subjected to correctional facilities for youths in the US, we assume that individuals are born in our economy at eighteen. We set the same discount factor as Fella and Gallipoli (2014), which falls between 0.989 used by İmrohoroğlu et al. (2004) and 0.76 found by Mastrobuoni and Rivers (2017). We run a Mincer regression on age, squared age, years of schooling, race and gender on the Current Population Survey (CPS) of 2004 in order to obtain the effect of the age on the productivity. The coefficients for age and age squared are 0.068 and -0.00128. The age from which the problem of the agent becomes stationary is chosen such that the productivity gain is 0, which happens at 45 years old, so $\widehat{T} = 27$. Due to very low yearly hazard rates in the US until 45 years old (lower than 0.3%), the probability of death for individuals until this age (corresponding to age $\widehat{T}$ in the model) is set to be zero, thus $\delta_t = 0$

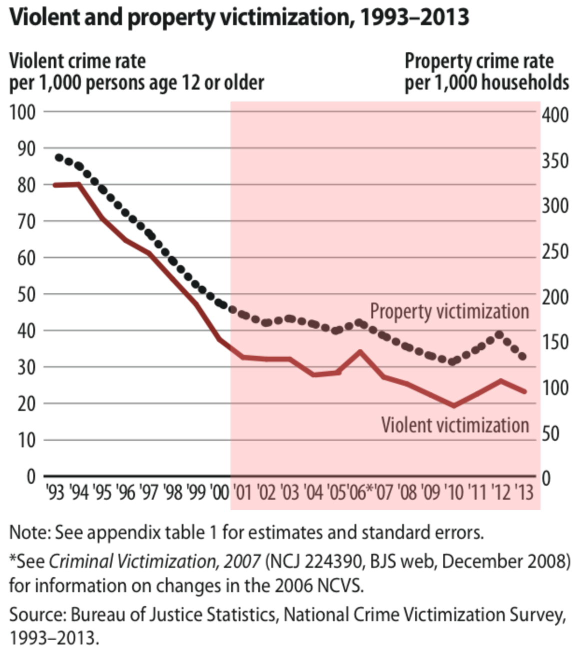


Figure 3: The crime rate becomes relatively steady between 2001 and 2015.

for $t \leq \widehat{T} - 1$. The death probability δ for agents older than T is calibrated with other remaining parameters.

We consider the productivity distribution of agents at 18 years old as lognormal with parameters (μ_w, σ_w). The Survey of Inmates in State and Federal Correctional Facilities, 2004, provides information on the monthly earnings from legal activities of prisoners the month prior incarceration. We estimate the parameters of the lognormal as $\widehat{\mu}_w = 4.78$ and $\widehat{\sigma}_w = 0.934$ (in hundreds of dollars), so the average monthly income of a 18 years old agent with propensity to commit crime is around 1600 dollars of 2004.

We turn our attention now to parameters of incarceration. We obtain the time served by convicted individuals released between 2001 to 2013 from the National Corrections Reporting Program (NCRP). Using a maximum likelihood procedure, $\widehat{\mu}_{2004}$ is estimated as 0.404, so agents spend, in average, 17.6 months in prisons for property crimes, close to 16 months from Engelhardt et al. (2008). Grogger (1995) finds that serving prison/jail results in a 30% reduction of incomes for individuals until 25 years old, when compared to similar individuals that were not incarcerated. Since average time in prison is 1.58 years, the yearly difference is 18%. Part of this difference is due to the depreciation of productivity in the legal sector of individuals that served prison, but the rest is due to gains of productivity of peers who remained free. Since the average rate of gains on income until 25 years old per year is around 5%, we set the depreciation of income per period in prison as $\widehat{\theta} = 1 - (0.18 - 0.05) = 0.87$.

According to the NCVS, 15 million property crimes were committed in 2004. Combining this survey, which also provides how the types of property crimes are distributed, with the estimates of the average market value of the transferred property and the deadweight loss from Cohen (2000), we set $\widehat{z}$ to 930 and $\widehat{d^V}$ to 600, both in dollars of 2004 per crimes. The value of $\widehat{d^V}$ can be seen as a lower bound for its actual value, since his methodology takes into account the costs of the crime ex post, excluding other costs such as the fear or efforts to avoid being victimized.

The average cost per prisoner per year $\widehat{\kappa}$ is taken from the US Bureau of the Census' Criminal Justice Expenditure and Employment Extract Program (CJEE) of 2004. Dividing the total expenditures on state and federal prisons by the total number of prisoners, we get $\widehat{\kappa} = 25,000$ dollars per prisoner per year. This value is between 30 thousand dollars found in the article "The price of Prison" from VERA institute and 23 thousand dollars used in Engelhardt et al. (2008).

The expenditures on apprehension technology devoted to detect and apprehend property property crimes k is a fraction h of 88 billion dollars, the total expenditures on police protection according to the CJEE of 2004. We assume that h is close to the fraction of new court commitments for property crimes, given by 27.1% and the fraction of prisoners' admissions for property crimes, which amount to 25%. We set $h = 25\%$, so $\widehat{k}_{2004} = 22$ billion dollars.

3.2 Calibrated Parameters

The remaining parameters to be calibrated are: the income-equivalent of being in jail b, the total dishonest population M_0, the average number of opportunities per year ν, the probability of giving up criminal activities, the apprehension probability parameters ζ_1, ζ_2, and the age-dependent cost incurred by a criminal for each crime. We define the cost from 18 to 24 years old as $d^C(0)$, from 25 to 34 years old as $d^C(1)$ and for more than 35 years old as $d^C(2)$.

We benchmark the model to match 9 different moments related to crime. Four of these moments are the number of prisoners in 2004 by age group, below 25 years, between 25 and 34 years, between 35 and 44 years and above 45 years. Two additional moments are the total number of property crimes in 2004 obtained from the NCVS of the same year and the proportion of individuals between 18 and 24 years old committing crimes, taken from the National Longitudinal Survey of Youth 1997 (NLS97). We also use the results of Abrams and Rohlfs (2007) which found that prisoners are willing to pay around a thousand dollars of 2004 to remain 90 days in freedom[14]. The last moments are the elasticities of the crime rate

[14]Let ρ be the average willingness to pay to avoid be sentenced, which is an average is taken on all apprehended agents. We define $N^E = \sum_{t=0}^{T-1} N_t^E + N_{T+}^E$ and consider the definitions of S_{T+}^E and N_{T+}^E given in Appendix C. Then

$$\begin{aligned}\rho &= \frac{1}{N^E p(v,k)} \left(\sum_{t=0}^{T-1} \int_0^{w_t^*} (U_t^F(w) - U_t^P(w)) m_t(w) \mathrm{d}w + \sum_{t=T}^{\infty} \int_0^{w_t^*} (U_t^F(w) - U_t^P(w)) m_t(w) \mathrm{d}w \right) p(v,k) \\ &= \frac{1}{N^E} \left(\sum_{t=0}^{T-1} \int_0^{w_t^*} (U_t^F(w) - U_t^P(w)) m_t(w) \mathrm{d}w + (a^F - a^P) N_{T+}^E + (b^F - b^P) S_{T+}^E \right).\end{aligned}$$

on both police expenditures and prison population, taken respectively from Levitt (2002) and Levitt (1996). Table 1 presents the results of this procedure of calibration. To put some

Parameter	Value
$\widehat{M_0}$	385,000
$\widehat{\nu}$	9.8 opportunities per year
$\widehat{\zeta}_1$	$1.1(\text{U\$ tr})^{-1}$
$\widehat{\zeta}_2$	1.3×10^{-6}year
$\widehat{b}$	9500 U$ per year
$\widehat{d^C}(0)$	60 U$ per crime
$\widehat{d^C}(1)$	840 U$ per crime
$\widehat{d^C}(2)$	880 U$ per crime
$\widehat{\delta}$	0.33

Table 1: Parameters obtained from the calibration process

parameters of Table 1 in perspective, $\widehat{M_0} = 385000$ and $\widehat{\delta} = 0.33$ imply that, in 2004, 9% of young individuals (from 18 to 24 years old) and 5% of the entire adult population in the US contemplate the possibility of committing crimes while the shares of engagement in crime for those two subpopulations are respectively 2.3% and 0.8%. The monthly equivalent-income of being in prison is 790 dollars, which means that less than 0.3% of the young individuals that are free would be better-off in prison.

3.3 Calibration performance

Our calibration procedure does a good job at matching the targeted moments, as shown in Table 2. The elasticity of crime on police spendings is the moment that deviates the most from our target and yet the relative difference does not exceed 10%. This difference of 0.05 is considerably smaller than 0.235, the variance of the elasticity estimated in Levitt (2002).

We assess the performance of the model for some non-targeted moments. Given the values of $\widehat{\nu}$ and $\widehat{\zeta}_1$, conditional on committing at least one crime, individuals engaged in crime commit on average, $(1 - e^{-\widehat{\nu}\widehat{p}})/\widehat{p} = 8.7$ crimes per year. The equivalent quantity obtained from the NLSY97 is 7.4. Since individuals are self-reporting their criminal activities for the NLSY97, the difference between our model and real data would be compatible to the belief that individuals are inclined to underreport the number of crimes they have committed.

We also compare the age profile of new admissions to jails/prisons for property crimes in the model with the equivalent quantity obtained from data. To do this, we use the

The right-side hand can be numerically calculated and depends on the parameters of the system.

Moment	Value benchmark	Value model
Level of crime	15 mi	15 mi
Proportion criminals < 25	2.3%	2.3%
Value of 90 days freedom	1000 $	1000 $
Prisoners < 25	143,000	143,000
Prisoners 25-34	153,000	153,000
Prisoners 35-44	134,000	134,000
Prisoners > 45	54,000	54,000
Elasticity crime - police spendings	-0.50	-0.55
Elasticity crime - prison population	-0.26	-0.27

Table 2: Comparing the targeted moments from data with the moments in the model

Sourcebook of Criminal Justice Statistics 2003, which reports the number of arrests by age and type of crime. This source is particularly interesting since it is independent from all other sources we use. To compare data from this source with our model, we assume that the probabilities of conviction and incarceration are independent of the age of the criminal. Results are presented in Table 3.

Age	Age group	Data	Model
18-24	1	40.4	39.4
25-34	2	26.1	22.3
35-44	3	22.0	27.4
> 45	4	11.3	10.9

Table 3: Share of new admissions to jails/prisons for property crimes by each age group, data versus model.

The model fits will the data on new admissions to jails/prisons by age group. The main qualitative difference is that, in the model, the share of criminals in age group 3 is higher than the share of criminals in age group 2, whereas we observe the opposite pattern in data. This inversion happens because, in the model, individuals in age group 3 discount the future more than individuals in age group 2. This additional discounting more than compensates the higher productivity and higher cost to commit crimes faced by individuals of age group 3.

4 Socially optimal public safety policies

This section presents the results of our policy experiments. With our model we can obtain, for any safety policy, the response of the economy and the social loss caused by crime and fighting-crime policies. As defined above, the social loss is the gap between a reference level of aggregate consumption—in a counter-factual situation in which the cost of committing each crime is infinity—and the actual aggregate consumption.

4.1 Procedures for the optimization and results

There are two general concerns about policy recommendations. First, due to political/administrative costs, the feasibility of a policy change depends critically on the magnitude of the changes. Second, because numerical calculations are subject to random approximation errors, we can conclude that policies that produce tiny differences in the social loss are socially equivalent. Thus, from the set of policies socially equivalent to the numerical solution of Equation (4), we pick and propose the one closest to the benchmark policy.

To numerically solve Equation (4), we use values of k ranging from 0 to 50 \$bn in increments of 0.1 \$bn and values of μ ranging from 0 to 1 in increments of 0.01. For policy recommendations, we consider two policies as being socially equivalent if their social loss difference is lower than 0.1 \$bn[15]. For policies that are socially equivalent, we choose the one with the smallest deviation in the average sentence length relative to the benchmark.[16] We find that the predicted socially optimal public safety policy is $(\widehat{k}^{**}, \widehat{\mu}^{**}) = (14.7 \text{ U\$ bn}, 0.39)$. Other variables of interest are presented in Table 4.

	Total	Δ% of benchmark
Average time in prison	18.7 mth	5.8
Expenditures on police	14.7 $bn	-33.2
Expenditures on PSP	25.6 $bn	-24.9
Number of crimes	19.1 mi	27.3
Number of prisoners	435,300	-10.1
Total loss	53.3 $bn	-4.6

Table 4: Variables of interest at the social optimum.

The policy recommendation is to cut police expenditures by approximately a third. This policy increases the crime rate by 27% and decreases the number of prisoners by 10% relative

[15]This value is enough small compared to typical values of social loss generated by our model. Particularly the social loss of 2004 estimated at 55.9 $bn.

[16]Conditional on this deviation being equal for two socially equivalent policies, we consider deviations in police expenditure from the benchmark.

to the values of 2004. These changes lead to a net social welfare gain of 2.6 billion dollars each year using the monetary metric.

It is worthwhile to discuss the divergence between our result and the result in Levitt (1997), in which Levitt recommends an increase in the expenditures in police protection. First, Levitt considers violent crimes and property crimes jointly, while we consider only property crimes (with robbery being a property crime). Using the numbers of our paper to Levitt's reasoning would still lead us to propose an increase police expenditures. However, the net social gain per extra sworn officer per year would fall to \$10,000, instead of more than \$200,000 he found.[17] Second, while Levitt includes the stolen property as a part of the social loss in a property crime, we do not include it. This would reduce the social cost per criminal action to less than half of Levitt's estimation, which would invert the direction of his recommendation. Finally, Levitt's ignores the social gain obtained by having a smaller prison population in terms of prison costs, freedom deprivation and human capital depreciation.

4.2 A positive analysis of the law enforcement system

To allow for a better understanding of the mechanisms by which safety policies affect the social loss, we partition the social loss into three components. The first component is the public safety cost, defined as the tax burden necessary to finance the public safety system. The second is the direct crime loss, defined as the sum non-financial loss suffered by victims due to crimes and the cost paid by criminals to commit crimes.[18] The third component is the loss generated by productivity depreciation and freedom deprivation, which we call legal sector criminal loss.

Figure 4 (a) shows how each component of the social loss varies when k changes fixing the average sentence length at the value of 2004. As expected, the direct crime loss is decreasing in k while the public system cost is increasing. Notice that the legal sector criminal loss is non monotonic in k. To understand why, we recall that the social losses from human capital depreciation and freedom deprivation are increasing in the productivity of an individual. When k is low, the apprehension probability is also low, so criminals are well-off because they escape prison. In an intermediate region of k, there is still a moderately large number of agents with relatively high productivity committing crimes and being incarcerated, implying a high legal sector criminal loss. As we increase k beyond this intermediate range, the remaining agents engaged in crime or in prison are agents with low productivity, which imposes a low legal sector criminal loss.

In Figure 5, we show how some variables of the economy change *as we decrease* the

[17] An additional sworn officer prevents a loss of 19.4 property crimes, representing a social benefit of \$21,000 per year. An officer costs \$40,000 per year, so, supposing that at each \$4 in police protection only \$1 is devoted to property crimes (an assumption we use), fighting property crimes costs \$10,000 per year per officer.

[18] Notice that if $d^V = 0$ and $d_t^C = 0$, regardless of the average financial transfer, the direct crime loss would be 0 and the optimal public safety policy would imply no spendings with safety policy. The financial transfers caused by crimes are included in this component, but it amounts to 0 since does not change the aggregate consumption—the criminal consumes the transfer in the place of the victim.

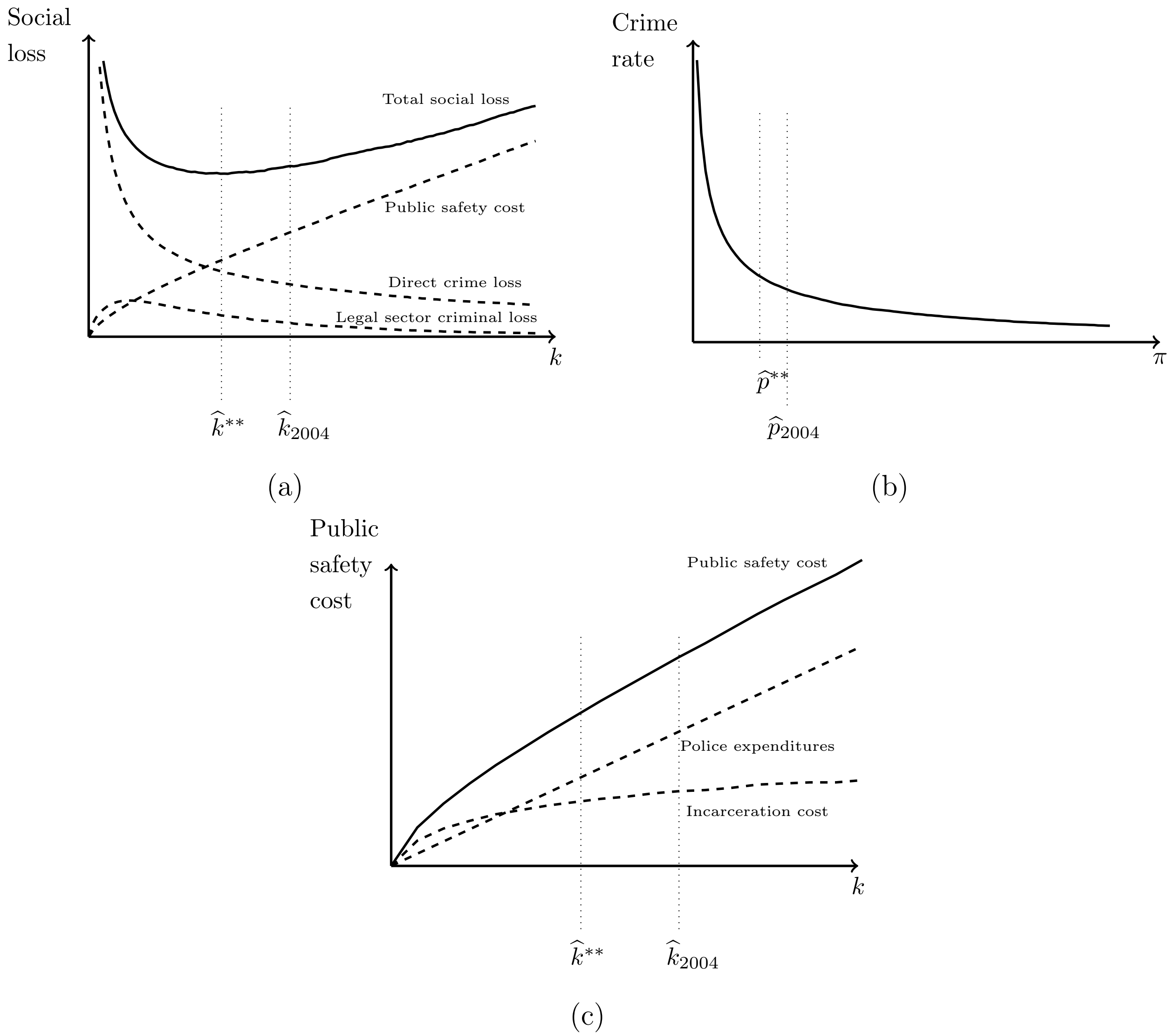


Figure 4: The behavior of some variables as k varies keeping the average sentence length at the value of 2004. (a) Components of the social loss and the total social loss (b) Crime rate as function of the apprehension probability. (c) Components of the public safety cost in incarceration cost and police expenditures.

severity of punishment fixing police expenditures at its optimal value. We can see that regardless of strengthening law enforcement via an increase in the severity of punishment or certainty of punishment, the components of the social loss follow the same patterns.

Figure 5 (b) shows that the reduction of the crime rate as the sentence length increases implies a smaller flow of criminals going into prison, but increasing the sentence length also decreases the flow of prisoners being set free. Figure 5 (c) shows that the net flow of prisoners increases with the sentence length, leading to a higher number of prisoners.

Maybe the most relevant difference between Figure 4 (a),(b) and Figure 5 (a),(b) is that a low value of μ is more effective in decreasing the crime rate than a high value of k. This difference is explained by the fact that some individuals with very low productivity will continue to commit crimes unless they are in jail, in other words, the main channel of fighting-crime policies when the safety policy is harsh is through incapacitation.

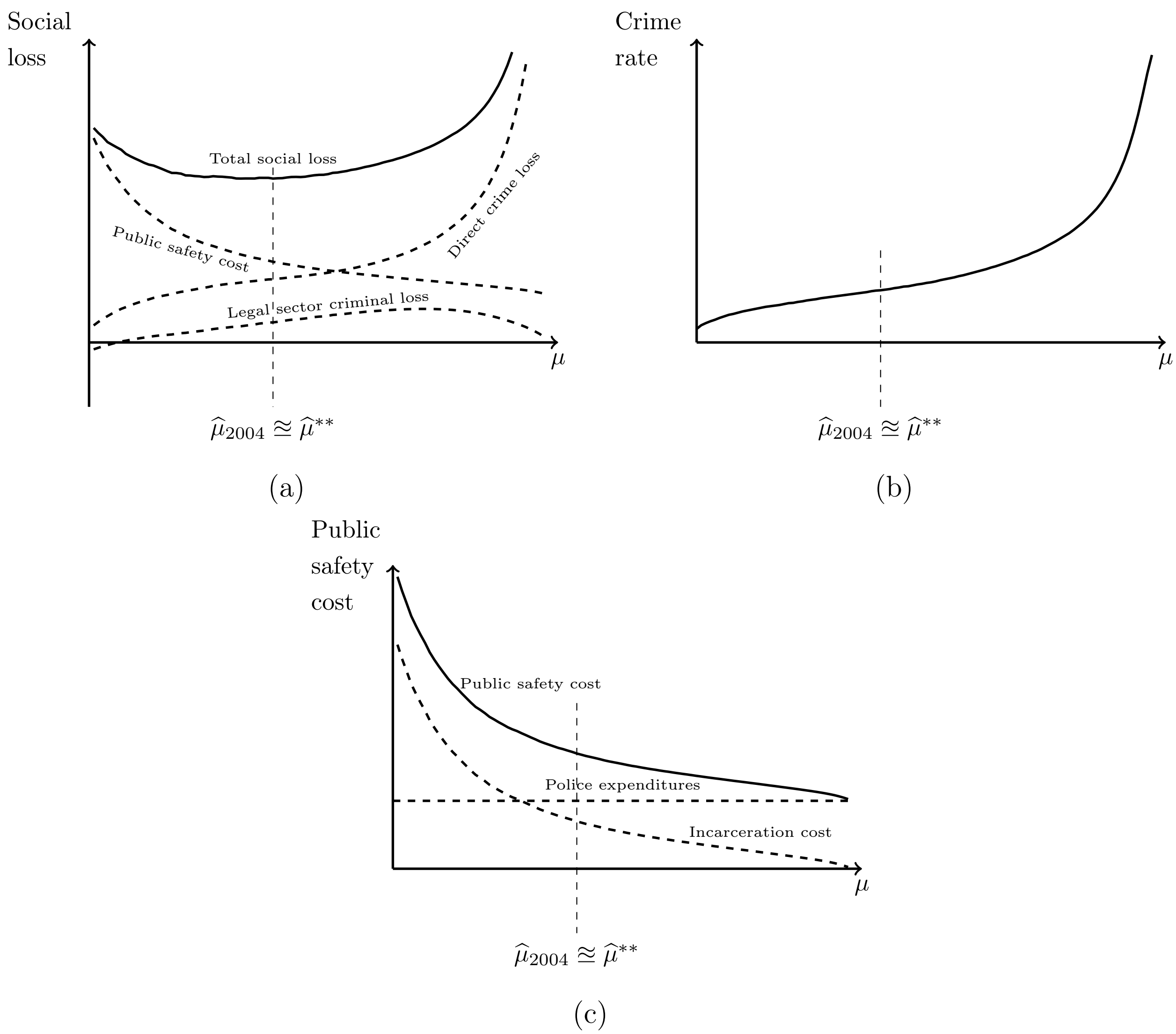


Figure 5: The behavior of some variables as μ varies keeping the average sentence length at the optimal level. (a) Components of the social loss and the total social loss (b) Crime rate as function of the apprehension probability. (c) Components of the public safety cost in incarceration cost and police expenditures.

4.3 Explaining the determinants of the policy recommendation

Our model predicts that the optimal safety policy would imply an important reduction in the police expenditures and a small reduction in the average sentence length and Figures 4 and 5 help us to understand why. Figure 4 (a) shows that as k increases, the legal sector criminal loss decreases. This reduction is ultimately a consequence of the fact that agents still committing crime under high values of k care less about being incarcerated. This means that, as the legal sector criminal loss decreases, the policy maker has less leverage to deter criminals. This behavior is displayed in Figure 4 (b), where the crime rate is shown as a function of the apprehension probability π. From this figure we can see regions where the marginal reduction of crime with π is high and where this marginal reduction is low. Because the number of prisoners is increasing in k (see Figure 4 (c)), the total public safety cost grows at least linearly in k, this makes the marginal social return negative whenever the marginal reduction of crime is low. The safety policy setting of 2004 falls exactly in the region where the response of criminal behavior to changes in the police expenditures is too inelastic and thus decreasing k improves the effectiveness of the public safety system. Regarding the severity of punishment, we can see that from Figure 5 (a) the social loss at $\widehat{\mu}_{2004}$ is in a relatively flat region, thus any change would produce only negligeable social gains.

Table 5 presents a quantitative breakdown of the social gains when moving from the policy of 2004 to the optimal policy. We highlight three main points about this table. First, the deciding factor for the police recommendation change is the high ineffectiveness of the police force relative the spendings with it at the 2004 safety policy. Second, although the optimal policy implies 10% reduction of the prison population, the social loss due to freedom deprivation increases 0.7 \$bn. This occurs because, as the apprehension probability decreases, agents with higher productivity commit crimes, changing the productivity profile in prison. In other words, changing to the optimal policy, who is in prison—rather than how many are in prison—is the responsible for increasing the loss due to freedom deprivation. Perhaps the most striking result is that the social loss due to crime increases at the optimal safety policy, which is counter-intuitive in the sense that the crime-fighting aspect of safety policies is usually associated to its effectiveness in reducing crime.

In the following two sections we try to accommodate, in two distincts ways, the idea that the crime rate should play a more central role in the social welfare measure. One is by imposing the restriction that no optimal policy should increase the current crime rate. The other is by using social welfare measures that consider the property stolen as a component of the social loss.

4.4 Optimal public safety policies under political constraints

Regardless of the improvement on the social welfare, policymakers may face political resistence if they propose a safety policy that increases the crime rate. In the same way, it may be hard to propose and implement a policy that can only be financed through an increase of the tax burden. If some of these constraints plays a decisive role in the implementation

All sources	2.6
Public Safety	
Total	8.5
Police	6.3
Prison	1.2
Crime	−3.8
Criminals	
Total	−2.1
Freedom deprivation	−0.7
HK depreciation	−1.4

Table 5: Welfare gain breakdown, in \$bn.

of safety policies, it is necessary to use second best solutions and this section is devoted to discuss the implications of those restrictions.

To help with this analysis, we consider Figure 6, which illustrates some level curves of social welfare, crime rate and public safety cost. Each point of the μk plane represents a possible safety policy. The black curves are social welfare indifference curves. Notice that these iso-welfare level curves are closed, indicating that for each average sentence length (resp. police expenditures) there are two values of police expenditures (resp. two values of average sentence lengths) resulting into the same social loss, one value where the public safety cost is low and the direct crime loss is high and the other value inverting the main source of social loss. The blue curve represents the set of policies with the same yearly expenditures on public safety as the one of 2004 and the policies in the blue region are attainable spending less on public safety. The red curve is the set of policies generating a crime rate equal to the one of 2004 and the policies in the red region are the ones that generate a crime rate smaller than the one of 2004. As expected regions of weak law enforcement are compatible with low public safety cost and high crime rate.

This figure also displays the optimal policy the 2004 safety policy $(\widehat{\mu}_{2004}, \widehat{k}_{2004})$, the unconstrained optimal policy (μ^{**}, k^{**}) and the socially optimal policy constrained to a crime rate reduction, $(\bar{\mu}, \bar{k})$. Thus (μ^{**}, k^{**}) is attainble with the 2004 budget, but the crime rate is higher than in 2004 with the optimal policy.

The intuition behind the position of the point $(\bar{\mu}, \bar{k})$ is as follows. From Figure 6 we see that a vertical displacement from $(\widehat{\mu}_{2004}, \widehat{k}_{2004})$ crosses more iso-welfare curves then a horizontal displacement does. This is compatible with the conclusion in Section 4.3 that, at the benchmark policy, the marginal return of decreasing the k is higher than the marginal return of changing μ. Being constrained to not increase the crime rate, the social planner would still decrease k controlling the crime rate by simultaneously increasing the average

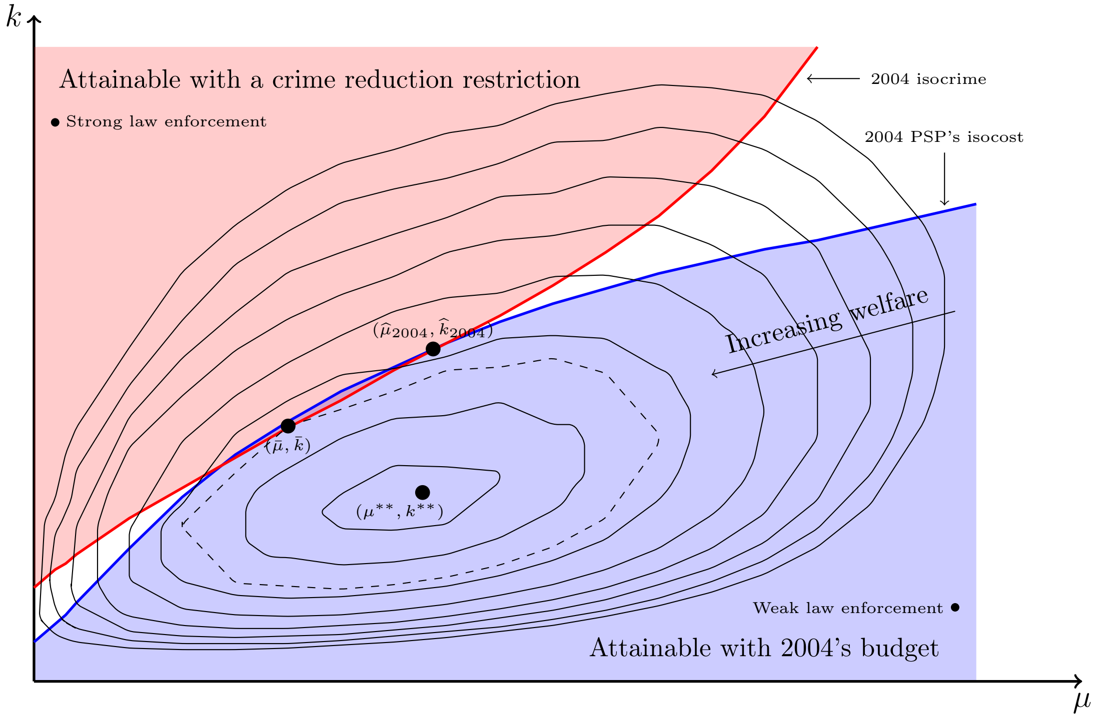


Figure 6: Contour lines and the political constraints of tax collection and crime level. The red region represents the set of policies leading to a crime reduction. The blue region is the set of policies leading to a tex-burden reduction.

sentence length. This pushes the social planner to move diagonally (left-down) the safety policy.

We numerically calculate the optimal solution respecting the constraint of not increasing the crime rate, given by $(\bar{\mu}, \bar{k}) = (0.29, 17.4)$, so the policy recommendation is to increase the severity of punishment and to decrease the police expenditures.[19] Other outcomes of this policy are presented in Tables 6 and 7.

	Total	Δ% of benchmark
Average time in prison	29.4 mth	65.5
Expenditures on police	17.4 $bn	-20.1
Expenditures on PSP	32.8 $bn	-3.8
Number of crimes	15.0 mi	0.0
Number of prisoners	615,300	27.1
Total loss	54.2 $bn	-3.0

Table 6: Variables of interest at the constrained social optimum.

All sources	1.7
Public Safety	
Total	1.3
Police	4.6
Prison	−3.3
Crime	−0.3
Criminals	
Total	0.7
Freedom deprivation	1.1
HK depreciation	−0.4

Table 7: Welfare gain breakdown with optimal policy under a crime reduction restriction, in $bn.

[19]The numerical procedure used to find the constrained optimum is the same one we applied in the unconstrained case. In addition, we allow the crime rate to be as large as 15.1 mi.

4.5 Other social welfare metrics and robustness

We incorporate elements of fairness and responsibility to this model. We introduce two alternative metrics for the social welfare and to make language more precise we name no responsibility-sensitive (NRS) the social welfare metric presented in Section 2. Finding the optimal policies for different welfare metrics and different values of h—the proportion of the total police expenditures devoted to property crimes, set in the calibration section at 0.25—provides us, at the same time, a robustness test of our policy recommendation in Section 4.1 and a better understanding of the way the welfare metric choice impact the policy recommendation.

We consider a fairness normative principle that excludes the consequences of a crime to a criminal from the social welfare. Under this principle, the total value of involuntary property transfers, defined as $L^{\text{prop}}(k,\mu)$, is view now as a social loss. This quantity is expressed in our model by $zv^*(k,\mu)(1-p(v,k))$. In the same way, costs paid by criminals to commit crimes, prisoners' freedom deprivation and prisoners' productivity depreciation are no longer considered as social losses. We name strictly responsibility-sensitive (SRS) the social welfare metric under this fairness principle, and the social loss denoted as $L^{\text{SRS}}(k,\mu)$, is given by

$$L^{\text{SRS}}(k,\mu) = L^{\text{pol}}(k,\mu) + L^{\text{keep}}(k,\mu) + L^{\text{crime}}(k,\mu) + L^{\text{prop}}(k,\mu).$$

The direct crime loss under this metric is the term $L^{\text{crime}}(k,\mu) + L^{\text{prop}}(k,\mu)$.

One can argue that the depreciation of productivity is actually an undesired side effect—a negative rehabilitation—of incarceration. Under this normative consideration, the productivity loss of free agents, denoted by $L^{\text{work}}(k,\mu)$, is considered a component of the social loss. We name responsibility-sensitive (RS) the social welfare metric implied by this argument and we denote the social loss in this case by $L^{\text{RS}}(k,\mu)$ and loss is expressed as

$$L^{\text{RS}}(k,\mu) = L^{\text{pol}}(k,\mu) + L^{\text{keep}}(k,\mu) + L^{\text{crime}}(k,\mu) + L^{\text{prop}}(k,\mu) + L^{\text{work}}(k,\mu).$$

The term L^{work}, the legal sector criminal loss under the RS metric, is evaluated in the Appendix C.

We calculate the optimal policy for various values of h and the three metrics of social welfare proposed in this paper.[20] Figure 7 presents the optimal policies. The policy recommendation is relatively robust across metrics and values of h. The optimal average sentence length ranges from 17.2 to 24 months and the optimal police expenditures from 13.1 to 17.2 \$bn. Qualitatively, the policy recommendation in 13 out of the 15 scenarios is to reduce the police expenditures—the exceptions being $h = 0.17$ and $h = 0.19$ under the SRS metric—and in 14 out of 15 scenarios the policy recommendation is to increase the average sentence length.

There are some patterns we observe across distinct welfare metrics. First, moving from SRS (green path) to RS (red path) results in a law enforcement strengthening for the optimal policy. A harsher law system under the RS metric may seem surprising since the criminals' welfare has more impact under the RS metric. However, a harsher law system deters agents

[20]We apply the numerical procedure used to find the optimum in section 4.1.

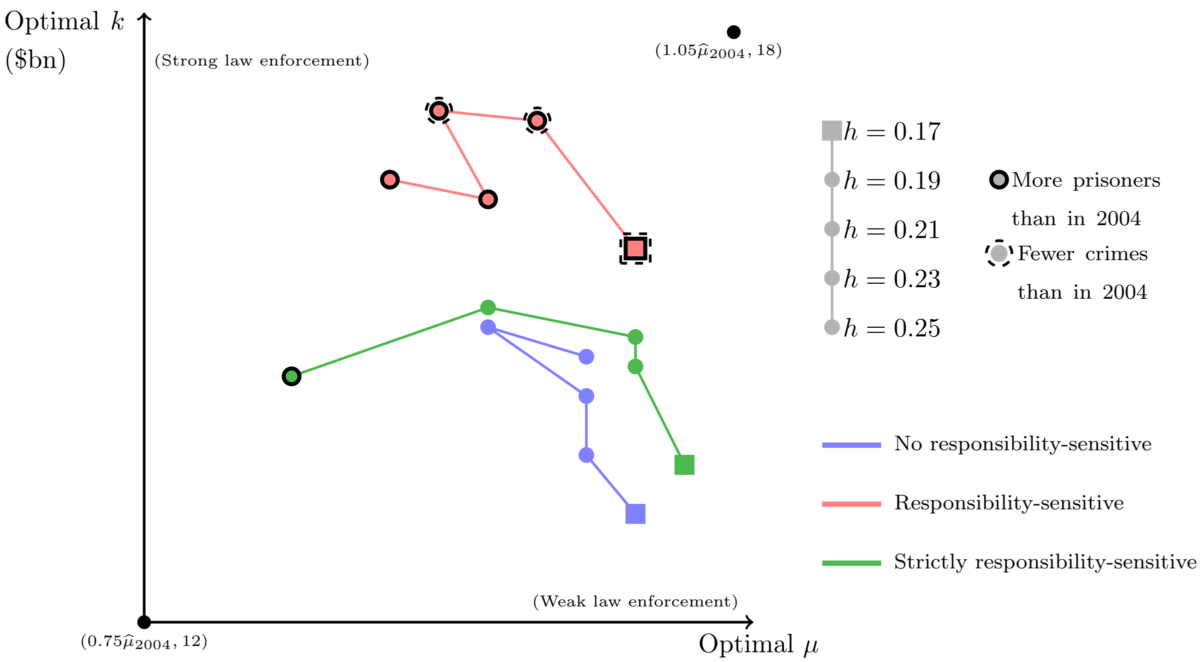


Figure 7: Global optima for various values of h and definitions of social welfare.

with relatively high productivity to commit crimes, which decreases the component loss L^{work} (only present in the RS metric). Second, we also observe that changing from NR (blue path) to SRS (green path) usually moves, by small magnitudes, the optimal police to a lower sentence and a higher apprehension probability. The overall effect of this change, for each h, reduces the crime rate, being compatible with the preponderant weight direct crime loss has under the SRS metric.

4.6 Marginal effects decomposition

The literature of crime is interested in the decomposition of the effects of safety policies on crime rates. The general understanding is that the marginal effect of changing the policy can be decomposed in incapacitation, deterrence and rehabilitation effects. We consider that none of these effects can explain the variations on the crime rate due to excessive load on police units, and we call this fourth effect *load effect*.

Appendix E presents the definition of each effect we deal in this section. Three points on these definitions deserve attention. First, we use the short term *marginal incapacitation effect* to denote the marginal incapacitation effect of a policy change on the crime rate (the analogous holds for rehabilitation, deterrence or load). However, it is possible to define, in a similar way, marginal effects on any other variable—such as number of prisoners or social loss. Second, to calculate the crime rate change, we look only at the stationary states after and before the policy change. Third, suppose that each policy leads to exactly one stationary equilibrium. In this case, the *total* incapacitation effect can be calculated by integrating the marginal changes through any path (in the sentence length/police expenditures plane) leading the initial policy to the changed policy.

We illustrate the intuition behind the definitions of the marginal effects using small changes in the police expenditures k. Following a small increase in k, the crime rate at the new stationary equilibrium is attained. The variation of the crime rate at the new equilibrium is the result of the combination of: (i) new beliefs in the number of police patrols (but old beliefs on police effectiveness); (ii) a new profile of productivity changing the mass of agents below the old cut-off productivity; (iii) a distinct number of police patrols in the streets (with their old effectiveness); and (iv) a different effectiveness of each police patrol. Isolating each one of these effects, we obtain, respectively, deterrence, rehabilitation, incapacitation, and load effects.

We numerically obtain the marginal effect decomposition of crime rate on k. Besides being an interesting exercise *per se*, this provides a better understanding of the interaction between safety policy and crime. Figure 8 displays the relation graphically.

The signs of the marginal effects are as expected. For some intermediate values of k, the marginal rehabilitation effect is positive. On the one hand, increasing k results in more arrested criminals among those not deterred by this higher k, which translates into a higher productivity depreciation for this group, pushing more agents under the cut-off productivity in older ages. On the other hand, the agents that were deterred will not suffer the productivity depreciation they would have for being in prison, pushing more agents above the cut-off productivity in older ages. The first effect dominates the second one when the marginal rehabilitation effect is positive. Finally, when k is small, changes on deterrence is basically driving the state of the economy. However, when k is high, the marginal deterrence effect becomes negligeable and all the reduction in the crime rate is achieved via incapacitation effect.

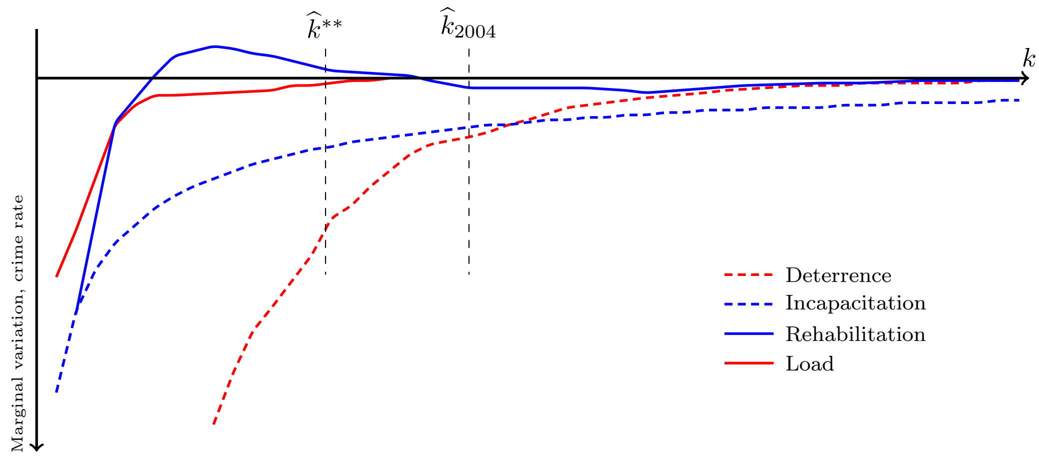


Figure 8: Marginal effect decomposition of the crime rate in k.

5 Conclusion

We have developed a framework that can be used to perform a comprehensive analysis of the impact of public safety policies on the economy. To do so, we have fully developed a life-cycle equilibrium model to capture the various non-trivial equilibrium responses and dynamic implications of crime-fighting policies. We have calibrated the model to 2004 US data and have numerically solved for the socially optimal policies under some different scenarios of political constraints and welfare metrics.

A key novelty of the model is the way we have dealt with the public security production function. Whereas previous papers in the literature have relied upon ad-hoc functional forms to represent this technology, we have given micro-foundations to a functional representation of the apprehension probability. The resulting apprehension probability function possesses all the properties the literature on crime has deemed important. In addition, we have established a link between the theoretical and the empirical literature by calibrating our apprehension technology with results from reduced-form estimations of the causal effect of police and prison population on crime.

Our results suggest current expenditures on policy protection in the US are too high, so spending less with police would increase the social welfare. We also find that the sentence length for property crimes seems to be close to its optimal value. As a consequence of these changes, we find that the optimal safety policy would lead to a lower prison population and a higher crime rate relative to 2004 levels. However, when considering a social loss metric that does not include some components of criminals' welfare resulting from criminal actions and punishment, the optimal policy would lead to a higher prison population and a (slightly) lower crime rate. The inversion of these results signals to the importance of the normative discussion about the objective of the criminal justice system. While the welfare metric presented in Section 2.6 has the appealing interpretation of representing the wealth, it does not consider the subjective value individuals give to feeling that they are in a fair society. This might be better captured in metrics including responsibility elements.

There are some limitations in the analysis that help us identify future avenues of research. First, due to practical challenges of calibration, we have restricted our analysis to property crimes. Including other types of crimes could capture the substititution effect across alternative of crimes. As a consequence, the resulting policy recommendations could be potentially more accurate and provide higher social gains. Second, micro-data could be used to push forward the understanding of the crime-detection mechanism, which would provide more accurate policy recommendations. Third, other components of human capital can be incorporated such as productivity gains in the criminal sector and the possibility of public expenditures on rehabilitation of incarcerated individuals. Finally, it would be interesting to incorporate a model for the justice system and the sentencing process to take into account, for example, the impact of the celerity of punishment and the possibility of wrong convictions.

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A Apprehension probability

We present details ommitted in the body of the article about the hypotheses of the apprehension model and how these hypotheses lead to Equation (1). For any arbitrary Borel set $A \subset \mathbb{R}^2$, we define $S(A)$ its area. Criminals can commit crimes at each point of a Borel set $R \subset \mathbb{R}^2$. By hypothesis, the number of police units is distributed as Poisson with parameter $\Lambda = \xi k^{\text{patrol}}$ and, conditional on the number of patrolling units, each unit is independently and identically distributed with a continuous distribution D over R, where D is chosen by the government. The game is played simultaneously.[21] According to Stoyan et al. (1987), the stochastic process of deployment of points is a Poisson point process with an intensity function $\lambda : R \to \mathbb{R}_+$, $x \mapsto D(x)\Lambda$ and we call this process Φ_D.

If a crime is committed at an arbitrary position $x \in R$, the probability $\pi(x; D)$ of detecting this crime is equal to the probability of having at least one police unit in the set $\mathbb{B}(x) = \{y \in \mathbb{R}^2 \text{ s.t. } \|x - y\| \leq r\}$ detecting the crime.[22] Thus we have that

$$\pi(x; D) = 1 - \mathbb{E}\Big(\prod_{y \in \Phi_D} (1 - q)\mathbb{1}_{y \in \mathbb{B}(x)}\Big)$$

and using the generatrix functional property of Φ_D we obtain[23]

$$\pi(x; D) = 1 - e^{-\int_{\mathbb{B}(x)} q\lambda(y)\mathrm{d}y}.$$

Recalling that each criminal wants to minimize the probability of having his crime detected, all crimes are committed in the set $\text{argmin}_{x \in R}\{\pi(x; D)\}$. The government seeks to maximize the proportion of crimes that are detected. It does so solving the problem

$$\underset{D \in C^0}{\text{argmax}}\big\{\min_{x \in R}\{\pi(x; D)\} | \textstyle\int_R D(y)\mathrm{d}y = 1\big\}$$

which is the same problem as[24]

$$\begin{gathered}\underset{D \in C^0}{\text{argmax}}\big\{\min_{x \in R}\{\textstyle\int_{\mathbb{B}(x)} D(y)\mathrm{d}y\}\big\} \\ \text{s.t.} \textstyle\int_R D(y)\mathrm{d}y = 1.\end{gathered}$$

[21] The same result holds if government was the first player.

[22] We ignore border effects, otherwise we should define $\mathbb{B}(x)$ as $\{y \in \mathbb{R}^2 \cap R \text{ s.t. } \|x - y\| \leq r\}$.

[23] The generatrix functional property of a Poisson point process states that, for a measurable function $u : R \to \mathbb{R}$, $\mathbb{E}\Big(\prod_{y \in \Phi_D} u(x)\Big) = e^{-\int (1-u(x))\lambda(\mathrm{d}y)}$.

[24] To see why, notice that

$$\begin{aligned}\max_{D \in C^0}\big\{\min_{x \in R}\{1 - e^{-\Lambda \int_{\mathbb{B}(x)} D(y)\mathrm{d}y}\} | \textstyle\int_R D(y)\mathrm{d}y = 1\big\} &= \max_{D \in C^0}\big\{1 - e^{-\Lambda \min_{x \in R}\{\int_{\mathbb{B}(x)} D(y)\mathrm{d}y\}} | \textstyle\int_R D(y)\mathrm{d}y = 1\big\} \\ &= 1 - e^{-\Lambda \max_{D \in C^0}\{\min_{x \in R}\{\int_{\mathbb{B}(x)} D(y)\mathrm{d}y\} | \int_R D(y)\mathrm{d}y = 1\}}.\end{aligned}$$

Because D is continuous and $\int_R D(y)\mathrm{d}y = 1$, any point $x \in R$ with $\int_{\mathbb{B}(x)} D(y)\mathrm{d}y > 1/S(R)$ would imply the existence of a point $x' \in R$ such that $\int_{\mathbb{B}(x')} D(y)\mathrm{d}y > 1/S(R)$. Thus, the unique solution of this problem is $D^* = 1/S(R)$ and we can write the apprehension probability as

$$1 - e^{-\xi k \int_{B(x)} \frac{1}{S(R)}\mathrm{d}y} = 1 - e^{-\zeta_1 k^{\text{patrol}}},$$

where $\zeta_1 = q\xi \frac{\pi r^2}{S(R)}$. With the distribution D^*, criminals are indifferent about the location to commit crimes. Thus, the only Nash equilibrium occurs when crime is uniformly distributed over R and the social planner chooses the distribution D^*. Any other strategy profile is ruled out as a Nash equilibrium by contradiction.

Until this point we express the apprehension probability as a function of k^{patrol}. We rewrite this relation to express it as a function of the crime rate v and the total police expenditures k. By hypothesis, the cost of each apprehension is α and the total number of apprehensions given v and k is $vp(v, k)$, so the total police expenditures are given by $k = k^{\text{patrol}} + \alpha vp(v, k)$. This means that we can write the apprehension probability as a function of k and v as

$$p(v, k) = 1 - e^{-\zeta_1(k - \alpha vp(v,k))} = 1 - e^{-\zeta_1 k + \zeta_2 vp(v,k)}.$$

Solving this functional equation, we obtain for all $v, k \geq 0$

$$p(v, k) = \begin{cases} 1 - \frac{W_L(\zeta_2 v e^{-\zeta_1 k + \zeta_2 v})}{\zeta_2 v} & \text{if } v > 0 \\ 1 - e^{-\zeta_1 k} & \text{if } v = 0. \end{cases}$$

We show that p respects several interesting properties that we expect from an apprehension probability function. Since W_L function is continuous and differentiable, p is continuous for $k \geq 0$ and $v > 0$ and differentiable in $k, v > 0$. The implicit relation of p guarantees the continuity also at $v = 0$. Differentiating implicitly $p(v, k) = 1 - e^{-\zeta_1 k + \zeta_2 vp(v,k)}$ with respect to k and v shows that p is strictly increasing in k, strictly concave in k and strictly decreasing in v. Differentiating implicitly the number of apprehensions $a(v, k) = v - ve^{-\zeta_1 k + \zeta_2 a(v,k)}$ with respect to k and v shows that a is strictly increasing in k and v. Using the implicit relation of p, we can show that: (i) $p(v, k) \to 1$ when $k \to \infty$; (ii) $p(v, k) \to 0$ when $k \to 0$; and (iii) $p(v, k) \to 0$ when $v \to \infty$.

B Existence of Equilibrium

We present here the guidelines to prove the existence of a stationary equilibrium and also the idea to compute it. Proposition B.1 and Corollary B.2 concern condition *(iii)* of the equilibrium as previously defined. Corollary B.3 incorporates conditions *(i)* and *(ii)*. Conditions *(i)*, *(ii)* and *(iii)* together allow us to express the belief in all variables of this economy in the stationary state as a function of the belief in a given level v. The restriction imposed by condition *(iv)* pins down v, since the belief in the crime rate of the equilibrium v^* has to be self-fulfilling (i.e., v^* is the fixed point). The fact that we can still obtain a non-empty set of equilibria after condition *(iv)* is respected is the statement of Theorem B.4. In the particular case of linear utilities and non-distortionary taxation, condition *(v)* does not have any impact on the individual or aggregate behavior and it is resumed by an accounting identity, respected by setting f accordingly. Condition *(v)* does affect, however, the level of welfare.

Let v be a belief in a crime rate. Define

$$\overline{\Pi}_t := \frac{(z(1-p(v,k)) - d_t^C)(1-e^{-p(v,k)\nu})}{p(v,k)},$$

$\widetilde{\beta}_t := \beta(1-\delta_t)$ and $\overline{p} := (1-e^{-p(v,k)\nu})(1-\mu)$. When $t \geq T$ we define $\overline{\Pi}_t := \overline{\Pi}$ and $\widetilde{\beta}_t := \widetilde{\beta}$. Notice that $\overline{\Pi}_t$ and $\overline{p}$ are functions of v. We also denote $V_t^F(w;v)$ and $V_t^P(w;v)$ as the value functions of an individual in freedom and in prison with productivity w, at age t as a function of v. The starting point is to notice that, when $t \geq T$, the problem faced by an individual is stationary. This allows to define the value functions $V_t^F(w;v)$ as $V^F(w;v)$ and $V_t^P(w;v)$ as $V^P(w;v)$ for $t \geq T$.

Proposition B.1. *For a given belief on crime rate v, there exists a cut-off productivity $w^*(v)$ for $t \geq T$ satisfying condition (iv) of the equilibrium such that an agent at age t engages in criminal activities if, and only if, his productivity is lower than $w^*(v)$. Moreover, there are closed-form expressions for $w^*(v)$, $V^F(w;v)$ and $V^P(w;v)$.*

Proof. Suppose that an agent with age w at age $t \geq T$ at some period engages in crime whenever $w \leq \tilde{w}$ (which does not necessarily solve the individual's dynamic program. Then, his (possible suboptimal) discounted utility when in freedom defined as $U_{\tilde{w}}^F$ and his discounted utility when in prison defined as $U_{\tilde{w}}^P$ respect the relations

$$U_{\tilde{w}}^F(w) = w - f + \overline{\Pi} + (1-\overline{p})\widetilde{\beta}U_{\tilde{w}}^F(w) + \overline{p}\widetilde{\beta}U_{\tilde{w}}^P(w),$$
$$U_{\tilde{w}}^P(w) = b - f + (1-\mu)U_{\tilde{w}}^P(\theta w) + \mu U_{\tilde{w}}^F(\theta w).$$

We start with the easier case where $w \leq \tilde{w}$. It happens that this functional system in this case has a solution (this can be checked by the guess-and-verify method). Let

$$\begin{aligned}
a^F &:= \frac{\overline{\Pi}(1-\widetilde{\beta}(1-\mu)) + b\widetilde{\beta}\overline{p}}{(1-\widetilde{\beta})(1-\widetilde{\beta}(1-\mu-\overline{p}))},\\
b^F &:= \frac{1-\widetilde{\beta}\theta(1-\mu)}{1+\widetilde{\beta}(1-\overline{p})(\widetilde{\beta}\theta(1-\mu)-1)-\widetilde{\beta}\theta(1-\mu(1-\widetilde{\beta}\overline{p}))},\\
a^P &:= \frac{\widetilde{\beta}\overline{\Pi}\mu + b(1-\widetilde{\beta}(1-\overline{p}))}{(1-\widetilde{\beta})(1-\widetilde{\beta}(1-\mu-\overline{p}))},\\
b^P &:= \frac{\widetilde{\beta}\theta\mu}{1+\widetilde{\beta}(1-\overline{p})(\widetilde{\beta}\theta(1-\mu)-1)-\widetilde{\beta}\theta(1-\mu(1-\widetilde{\beta}\overline{p}))}.
\end{aligned}$$

Then,

$$\begin{aligned}
U^F_{\tilde{w}}(w) &= a^F + b^F w,\\
U^P_{\tilde{w}}(w) &= a^P + b^P w.
\end{aligned}$$

Now we show the case $w > \tilde{w}$. An agent with productivity $w > \tilde{w}$ in freedom never engages in crimes, so $U^F_{\tilde{w}}(w) = w/(1-\widetilde{\beta})$. Suppose that this agent is in prison at t and he is released at $t' > t$. As a consequence of $\theta < 1$, he will have a productivity $w' > \tilde{w}$ in t' as long as

$$t' - t \leq \lceil \ln(\tilde{w}/w)/\ln(\theta) \rceil =: n(w).$$

Otherwise, his productivity drops to a value under $\tilde{w}$ and he goes to the regime of engaging in crimes. As a result of this dynamics, we have that

$$\begin{aligned}
U^P_{\tilde{w}}(w) = (1-\mu)^{n(w)-1}\widetilde{\beta}^{n(w)}((1-\mu)(a^P+b^P\theta^{n(w)}w)+\mu(a^F+b^F\theta^{n(w)}w))+b\left(\frac{1-(\widetilde{\beta}(1-\mu))^{n(w)}}{1-\widetilde{\beta}(1-\mu)}\right)\\
+\frac{\widetilde{\beta}\mu\theta w}{1-\widetilde{\beta}}\left(\frac{1-(\theta\widetilde{\beta}(1-\mu))^{n(w)-1}}{1-\theta\widetilde{\beta}(1-\mu)}\right).
\end{aligned}$$

For an arbitrary $\tilde{w}$ and a given v, the expressions of $U^F_{\tilde{w}}$ and $U^P_{\tilde{w}}$ are the solution of a system, rather then the solution of the individuals' dynamic program. Now, define $w^*(v)$ such that it solves $a^F + b^F w^*(v) = \frac{w^*(v)}{1-\widetilde{\beta}}$, explicitly given by

$$w^*(v) = \frac{[1+\widetilde{\beta}(1-\overline{p})(\widetilde{\beta}\theta(1-\mu)-1)-\widetilde{\beta}\theta(1-\mu(1-\widetilde{\beta}\overline{p}))][\overline{\Pi}(1-\widetilde{\beta}(1-\mu))+b\widetilde{\beta}\overline{p}])}{(1-\widetilde{\beta}(1-\mu-\overline{p})\widetilde{\beta}\overline{p}(1-\widetilde{\beta}\theta)}.$$

We can verify after some algebra that $b^F < \frac{1}{1-\beta}$, so $U^F_{w^*(v)}(w)$ and $U^P_{w^*(v)}(w)$ solve the dynamic program, i.e., $U^F_{w^*(v)}(w) = V^F(w;v)$ and $U^P_{w^*(v)}(w) = V^P(w;v)$. □

Then, we can show the next corollary.

Corollary B.2. *For a given belief on the crime rate v, there exists a sequence of cut-off productivities $w_t^*(v)$ for $t < T$ satisfying condition (iv) of the equilibrium such that an agent at age t engages in criminal activities if, and only if, his productivity is lower than $w_t^*(v)$. Moreover, there are recursive expressions for $V_t^F(w;v)$, $V_t^P(w;v)$ and $w_t^*(v)$.*

Proof. For a given v, we can find every $w_t^*(v)$ by backward induction. The first step is at age $T-1$. In this case, the dynamic program an agent at age with productivity w has to solve is

$$V_{T-1}^F(w;v) = w - f + \max\{\widetilde{\beta}_t V_T^F(\gamma_{T-1}w;v), \overline{\Pi}_t + (1-\overline{p})\widetilde{\beta}_t V_T^F(\gamma_{T-1}w;v) + \overline{p}\widetilde{\beta}_t V_T^P(\gamma_{T-1}w;v)\}.$$

with the following value function in prison

$$V_t^P(w;v) = b - f + (1-\mu)\widetilde{\beta}_t V_{t+1}^P(\theta w;v) + \mu\widetilde{\beta}_t V_{t+1}^F(\theta w;v).$$

Using the expressions of Proposition B.1, we see that $V_t^F(\cdot;v)$ is linear and continuous by parts and we can show that derivative of the first term of the maximand is higher than the derivative of the second term. As a consequence, we have the same type of behavior of agents at age over T, with a cut-off productivity $w_{T-1}^*(v)$ for individuals at age $T-1$. This cut-off productivity solves

$$\frac{\overline{\Pi}_{T-1}}{\overline{p}\widetilde{\beta}_{T-1}} = V^F(\gamma_{T-1}w_{T-1}^*(v);v) + V^P(\gamma_{T-1}w_{T-1}^*(v);v).$$

The value function of a free agent at age $T-1$ is given by

$$V_{T-1}^F(w;v) = \begin{cases} w + \overline{\Pi}_{T-1} + \widetilde{\beta}_{T-1}(1-\overline{p})V^F(\gamma_{T-1}w) + \widetilde{\beta}_{T-1}\overline{p}V^P(\gamma_{T-1}w;v) & \text{if } w \leq w_{T-1}^*(v) \\ w + \widetilde{\beta}_{T-1}V^F(\gamma_{T-1}w;v) & \text{if } w > w_{T-1}^*(v). \end{cases}$$

When this agent is in prison, his value function is given by

$$V_{T-1}^P(w;v) = b + (1-\mu)\widetilde{\beta}_{T-1}V^P(\theta w;v) + \mu\widetilde{\beta}_{T-1}V^F(\theta w;v).$$

The generalization for any age t follows the same line. So given the value functions at age $t+1$, we can show that there is a cut-off productivity solving the individuals' dynamic program (the functions are always continuous and linear by parts). $w_t^*(v)$ satisfies

$$\frac{\overline{\Pi}_t}{\overline{p}\widetilde{\beta}_t} = V_{t+1}^F(\gamma_t w_t^*(v);v) - V_{t+1}^P(\gamma_t w_t^*(v);v).$$

and the expressions for the value functions are

$$V_t^F(w;v) = \begin{cases} w + \overline{\Pi}_t + \widetilde{\beta}_t(1-\overline{p})V_{t+1}^F(\gamma_t w) + \widetilde{\beta}_t\overline{p}V_{t+1}^P(\gamma_t w;v) & \text{if } w \leq w_t^*(v) \\ w + \widetilde{\beta}_t V_{t+1}^F(\gamma_t w;v) & \text{if } w > w_t^*(v). \end{cases}$$

and, once more, the value function in prison is an affine combination of the value functions at $t+1$

$$V_t^P(w;v) = b + (1-\mu)\widetilde{\beta}_t V_{t+1}^P(\theta w;v) + \mu\widetilde{\beta}_t V_{t+1}^F(\theta w;v).$$

This completes the proof. □

Since we know the behavior of the agents as a function of v, how productivity evolve as a function of their state, and the initial distribution of productivity, we can find the measure of productivity at all ages. Suppose that we know the measure of agents that are free and the measure of agents that are in prison at $t-1$. The general principle to find the measure of agents with productivity w in the cohort t is to search for the agents at age $t-1$ that will end up with an productivity w at t and add all the incoming flows to obtain the total measure of agents with productivity w at age t. Since we have the measure of productivity in the cohort 0, we can use induction to find the measure of productivity for each cohorts.

To formalize this idea, we introduce the follow variables and functions. For a given crime rate and for each $s \in \{\tilde{E}, E, P\}$, let $m_t^s(w; v)$ be the measure of agents at age t with productivity w.[25] Then, we denote $M_t^s(w; v) := \int_0^w m_t^s(w'; v)\mathrm{d}w'$ with $M_t^s(\infty; v) := N_t^s(v)$. (so $N_t^E(v)$ is the mass of agents at age t engaging in criminal activities and $N_t^P(v)$ is the mass of prisoners at age t). Let also $v_t(v)$ be the number of committed crimes by agents at age t. Recall that agents at age 0 are in freedom, so $M_0^{\tilde{E}}(\cdot; v) + M_0^E(\cdot; v) = M_0 G_0$, $m_0^P(\cdot; v) = 0$ and that we can obtain $w_0^*(v)$ by Corollary B.2.

Corollary B.3. *For a given crime rate belief v, we can obtain recursive expressions for $m_t^F(\cdot; v)$, $m_t^P(\cdot; v)$ and $v_t(v)$ as functions of $m_{t-1}^F(\cdot; v)$ and $m_{t-1}^P(\cdot; v)$ satisfying conditions (i) and (iii) of equilibrium. Moreover, we can obtain a closed-form expression for $\sum_{t=T}^{\infty} v_t(v)$ as a function of $M_T^P(w; v)$ and $M_T^F(w; v)$.*

Proof. First part: obtaining the measures recursively. We use induction. Although there are no closed-form expressions for the measures, we can find recursive relations. For the age 0, recall that agents are free when they enter into the economy, so $N_0^P(v) = 0$, $N_0^E(v) = M_0 G_0(w_0^*(v))$ and

$$v_0(v) = \frac{(1 - e^{-\nu p})}{p} N_0^E(v) = \frac{\overline{p}}{(1-\mu)p} N_0^E(v)$$

Besides, $m_0^P(w; v) = 0$, $m_0^E(w; v) = M_0 g_0(w) \mathbb{1}_{[w \le w_0^*(v)]}$ and $m_0^{\tilde{E}}(w; v) = M_0 g_0(w) \mathbb{1}_{[w > w_0^*(v)]}$ and the mass of new incarcerated agents at this age is $(1 - e^{-\nu p^*})(1-\mu) N_0^E$

Consider now the measures for a generic t given $t-1$. There are two possibilities allowing an individual to be in prison at age t to have productivity w. The first one is that, at age $t-1$, he was free with productivity w/γ_{t-1} and he was engaged in crime, apprehended and incarcerated. The second one is that, at age $t-1$, he was in prison with productivity w/θ and he was not released from age $t-1$ to age t. Since a proportion $(1-\mu)(1-\delta_{t-1})$ of agents at age $t-1$ will remain in prison at age t, and a proportion $(1-\delta_t)\overline{p}$ of agents engaged on crime at age $t-1$ are prisoners at age t, we have that

$$m_t^P(w; v) = (1 - \delta_{t-1}) \left(\overline{p} \frac{m_{t-1}^E(w/\gamma_{t-1}; v)}{\gamma_{t-1}} + (1-\mu) \frac{m_{t-1}^P(w/\theta; v)}{\theta} \right).$$

We calculate $m_2^E(w; v)$ in a similar way. The following groups of agents at age $t-1$ start period t with age w: a proportion $(1-\delta_{t-1})$ of agents not engaged in crime with productivity

[25] Notice that, if an equilibrium exists with crime rate v^*, $m_t^s(w; v^*) = m_t^s(w)$.

w/γ_{t-1}; a proportion $(1-\delta_{t-1})(1-\overline{p})$ of agents engaged in crime with productivity w/γ_{t-1}; a proportion $(1-\delta_{t-1})\mu$ of agents in prison with productivity w/θ. So

$$m_t^E(w;v)+m_t^{\tilde{E}}(w;v) = (1-\delta_{t-1})\Big(\frac{(1-\overline{p})m_{t-1}^E(w/\gamma_{t-1};v)+m_{t-1}^{\tilde{E}}(w/\gamma_{t-1};v)}{\gamma_{t-1}}+\mu\frac{m_{t-1}^P(w/\theta;v)}{\theta}\Big).$$

Then, according to Corollary B.2, $m_t^E(w;v) = (m_t^E(w;v)+m_t^{\tilde{E}}(w;v))\mathbb{1}_{[w\leq w_t^*(v)]}$ and $m_t^E(w;v) = (m_t^E(w;v)+m_t^{\tilde{E}}(w;v))\mathbb{1}_{[w>w_t^*(v)]}$. From those differential relations, we obtain the integral ones. The total number of prisoners at age t is $N_t^P(v) = \int_0^\infty m_t^P(w';v)\mathrm{d}w'$ and the total number of agents engaged in crime at this age is $N_t^E = \int_0^\infty m_2^E(w';v)\mathrm{d}w'$. The total number of crimes committed by agents at age t is then given by

$$v_t(v) = \frac{\overline{p}}{(1-\mu)p}N_t^E(v)$$

and the total number of apprehensions is $\overline{p}N_t^E$.

Second part: finding the total number of crimes. The belief in the total number of crimes for a belief in a crime rate v is given by

$$\sum_{t=0}^{\infty} v_t(v) = \sum_{t=0}^{T-1} v_t(v) + \sum_{t=T}^{\infty} v_t(v).$$

The first term is straightforward from the first part of the proof. We analyse the second term. We refer to agents in prison of all ages $t \geq T$ as agents of age T^+. Let $A_0 := M_T^P(w^*(v))$ and $A_i := M_T^P(w^*(v)/\theta^i) - M_T^P(w^*(v)/\theta^{i-1})$ for $i \geq 1$, i.e., the mass of agents in prison at age T with productivity between w^*/θ^i and w^*/θ^{i-1}. Denote as $N_{T^+}^P(v)$ the total number of agents in prison of age T^+ and $N_{T^+}^E(v)$ the total mass of agents engaged in crime of age T^+. Define

$$x_0^M := \sum_{i=0}^{\infty} A_i[(1-\mu)(1-\delta)]^i$$

and

$$y_0^M := M_T^f(w^*) + \mu\sum_{i=1}^{\infty} A_i(1-\mu)^{i-1}(1-\delta)^i.$$

If an individual is in prison at period T, his productivity at this age is between $w^*(v)/\theta^i$ and $w^*(v)/\theta^{i-1}$ and it remains i periods in prison, then at $T+i$ this agent will be in prison with productivity lower than $w^*(v)$. So the quantity $A_i(1-\mu)^{i-1}(1-\delta)^i$ is the mass of agents released from prison at age $T+i$, with a productivity higher than $w^*(v)$ at $T+i-1$ and lower than $w^*(v)$ at $T+i$. It follows that x_0^M is the mass of all agents at age T^+, in prison, reaching productivities below w^* for the first time while y_0^M is the equivalent to x_0^M for agents in freedom.

Let x_τ^M be the mass of agents of age T^+ in prison that reached productivities below $w^*(v)$ for the first time τ periods after T and y_τ^M be the equivalent to agents in freedom. Because agents with productivity lower than $w^*(v)$ are engaged on crime, the dynamics of x_τ^M and y_τ^M is given by

$$\begin{aligned} x_{\tau+1}^M =&(1-\delta)\overline{p}y_\tau^M + (1-\delta)(1-\mu)x_\tau^M, \\ y_{\tau+1}^M =&(1-\delta)(1-\overline{p})y_\tau^M + (1-\delta)\mu x_\tau^M. \end{aligned}$$

We can analytically solve the general term of this first order homogeneous difference equation. The term $\sum_{\tau=0}^{\infty} y_\tau^M$ is a geometric progression series of the general term of the difference equation and we find the value of $N_{T^+}^E(v)$ noticing that $N_{T^+}^E(v) = \sum_{\tau=0}^{\infty} y_\tau^M$:

$$N_{T^+}^E(v) = \frac{(x_0^M + y_0^M)\mu}{\delta(\mu+\overline{p})} - \frac{\mu x_0^M - \overline{p}y_0^M}{[1-(1-\delta)(1-\mu-\overline{p})](\mu+\overline{p})}.$$

We can partition individuals in prison at age T^+ in two groups: the ones with age below $w^*(v)$, that amount to $\sum_{\tau=0}^{\infty} x_\tau^M$, and the ones with productivity above $w^*(v)$, that amounts to $\sum_{i=1}^{\infty} A_i \frac{1-[(1-\mu)(1-\delta)]^i}{1-(1-\mu)(1-\delta)}$. Thus, we can obtain $N_{T^+}^p(v)$:

$$N_{T^+}^P(v) = \sum_{i=1}^{\infty} A_i \frac{1-[(1-\mu)(1-\delta)]^i}{1-(1-\mu)(1-\delta)} + \frac{(x_0^M + y_0^M)\overline{p}}{\delta(\mu+\overline{p})} + \frac{\mu x_0^M - \overline{p}y_0^M}{[1-(1-\delta)(1-\mu-\overline{p})](\mu+\overline{p})}.$$

The total number of victims caused by this subgroup of agents is given by $v_{T^+}(v) = N_{T^+}^E(v)\frac{\overline{p}}{(1-\mu)p}$ and the proof is complete. □

Using the previous results, regularity conditions and the Intermediate Value Theorem, we can show the existence of a stationary equilibrium. We highlight this result in the following theorem.

Theorem B.4. *There is at least one stationary equilibrium.*

Proof. First, we notice that $0 \leq \sum_{t=0}^{\infty} v_t(v) \leq \overline{M}$, where $\overline{M}$ is the crime rate when the apprehension probability is 0, i.e., all dishonnest agents engage in crime. The general idea of the proof is to show that $\sum_{t=0}^{\infty} v_t(v)$ is continuous with respect to v. Since the image is compact, we can use the Intermediate Value Theorem to show that there is at least one fixed point. The proof is tedious and resumes to prove the following points, in this order: $w^*(v)$ is continuous on v (it is trivial to show using Proposition B.1); $V^F(\cdot; v)$ and $V^P(\cdot; v)$ are uniformly continuous (implying that $V^F(\cdot; v) - V^P(\cdot; v)$ is uniformly continuous); by (backward) induction, $w_t^*(v)$ is continuous in v; $N_t^s(v)$ is continuous for all $s \in \{\tilde{E}, E, P\}$ and all $t \in \{0, 1, \ldots\}$ (from standard results of the measure theory); $N_{T^+}^s(v)$ is continuous; $\sum_{t=0}^{\infty} v_t(v) = \sum_{t=0}^{T-1} v_t(v) + v_{T^+}(v)$ is also continuous. □

C Expressions for $L^{\mathbf{FD}}$, $L^{\mathbf{HK}}$, and $L^{\mathbf{work}}$

Expressions for $L^{\mathbf{FD}}$ and $L^{\mathbf{HK}}$

Define x_τ^S as the mass of productivities of agents of age T^+ in prison that reached productivites below w^* for the first time τ periods after T and y_τ^S be the equivalent to agents in freedom. Let $B_0 := \int_0^{w^*} w m_T^P(w)\mathrm{d}w$ and

$$B_i := \int_{\frac{w^*}{\theta^{i-1}}}^{\frac{w^*}{\theta^i}} w m_T^P(w)\mathrm{d}w,$$

i.e., B_i is the productivity mass of agents at age T in prison having productivity between w^*/θ^i and w^*/θ^{i-1}. Let $S_{T^+}^P$ the productivity mass of all agents in prison at age T^+ and $S_{T^+}^F(w^*)$ the mass of productivity of all agents engaged in crime at age T^+. Thus, we can express the initial masses of productivities as

$$x_0^S := \sum_{i=0}^{\infty} B_i[\theta(1-\mu)(1-\delta)]^i$$

and

$$y_0^S := \int_0^{w^*} w m_T^E(w)\mathrm{d}w + \mu \sum_{i=1}^{\infty} B_i \theta^i (1-\mu)^{i-1}(1-\delta)^i.$$

The main idea to find the mass of wages of agents of age T^+ is similar to the one of Corollary B.3 to find v_{T^+}. The main difference is that the mass of productivity of agents in prison also decreases due to depreciation of productivity in prison, so we have the following difference equation

$$\begin{bmatrix} x_{\tau+1}^S \\ y_{\tau+1}^S \end{bmatrix} = \underbrace{\begin{bmatrix} (1-\delta)(1-\mu)\theta & (1-\delta)\overline{p} \\ (1-\delta)\mu\theta & (1-\delta)(1-\overline{p}) \end{bmatrix}}_{:=A} \begin{bmatrix} x_\tau^S \\ y_\tau^S \end{bmatrix}.$$

Then, defining v_1^A and v_2^A as the eigenvectors of A associated, respectively, to the eigenvalues λ_1^A and λ_2^A of A and $P^A = [v_1^A\ v_2^A]$, we can show that $S_{T^+}^P$, the total mass of productivities of agents in prison of age T^+, and $S_{T^+}^E$, the analogous for agents in freedom with productivity below w^*, are given by

$$\begin{bmatrix} S_{T^+}^P \\ S_{T^+}^E \end{bmatrix} = P^A \begin{bmatrix} \frac{1}{1-\lambda_1} & 0 \\ 0 & \frac{1}{1-\lambda_2} \end{bmatrix} (P^A)^{-1} \begin{bmatrix} x_0^S \\ y_0^S \end{bmatrix} + \begin{bmatrix} \sum_{i=1}^{\infty} B_i \frac{1-[\theta(1-\mu)(1-\delta)]^i}{1-\theta(1-\mu)(1-\delta)} \\ 0 \end{bmatrix}.$$

Using this result and Corollary B.3, we can calculate the total loss of welfare inflicted on agents. The loss caused by freedom deprivation is

$$L^{\mathrm{FD}} = S_{T^+}^P - bN_{T^+}^P + \sum_{t=0}^{T-1} \left(\int_0^{\infty} w m_t^P(w)\mathrm{d}w - bN_t^P \right).$$

Defining

$$S^{\tilde{E}}_{T^+} := \frac{1}{\delta}\int_{w^*}^{\infty} w m^{\tilde{E}}_T(w)\mathrm{d}w + \theta(1-\delta)\mu\sum_{i=1}^{\infty} B_{i+1}\frac{1-(\theta(1-\delta)\mu)^i}{1-\theta(1-\delta)\mu},$$

the loss caused by depreciation of productivity is given by

$$\begin{aligned}L^{\mathrm{HK}} &= \frac{\prod_{t'=0}^{T-1}[(1-\delta_{t'})\gamma_{t'}]}{\delta}M_0\int_0^{\infty} wg_0(w)\mathrm{d}w - (S^E_{T^+}+S^P_{T^+}+S^{\tilde{E}}_{T^+})\\ &+\sum_{t=0}^{T-1}\left(\prod_{t'=0}^{t-1}[(1-\delta_{t'})\gamma_{t'}]M_0\int_0^{\infty} wg_0(w)\mathrm{d}w - \int_0^{\infty} w(m^{\tilde{E}}_t(w)+m^E_t(w)+m^P_t(w))\mathrm{d}w\right).\end{aligned}$$

Expression for L^{work}

The last term to evaluate is L^{work}. This loss is defined in Section 5.2 as the difference between a counterfactual total productivity of dishonest agents in freedom and their actual total productivity. Agents in the counterfactual situation use the decision rules of agents in the actual situation, but do not suffer productivity depreciation due to incarceration. We can calculate the counterfactual distributions until age T, defined as $\dot{m}^P_t(w)$, $\dot{m}^E_t(w)$, and $\dot{m}^{\tilde{E}}_t(w)$, in the same way we have obtained $m^P_t(w)$, $m^E_t(w)$, and $m^{\tilde{E}}_t(w)$ in Appendix B. So, for all $w \geq 0$, $\dot{m}^P_0(w) = 0$, $\dot{m}^E_0(w) = M_0 g_0 \mathbb{1}_{[w\geq w^*_0]}$, and $\dot{m}^{\tilde{E}}_0(w) = M_0 g_0 \mathbb{1}_{[w>w^*_0]}$

$$\dot{m}^P_t(w) = \frac{(1-\delta_{t-1})}{\gamma_{t-1}}\left(\overline{p}\dot{m}^E_{t-1}(w/\gamma_{t-1}) + (1-\mu)\dot{m}^P_{t-1}(w/\gamma_{t-1})\right)$$

and

$$\dot{m}^E_t(w) + \dot{m}^{\tilde{E}}_t(w) = \frac{(1-\delta_{t-1})}{\gamma_{t-1}}\left((1-\overline{p})\dot{m}^E_{t-1}(w/\gamma_{t-1}) + \dot{m}^{\tilde{E}}_{t-1}(w/\gamma_{t-1}) + \mu\dot{m}^P_{t-1}(w/\gamma_{t-1})\right).$$

Besides, $\dot{m}^E_t(w) = (\dot{m}^E_t(w) + \dot{m}^{\tilde{E}}_t(w))\mathbb{1}_{[w\leq w^*_t]}$ and $\dot{m}^E_t(w) = (\dot{m}^E_t(w) + \dot{m}^{\tilde{E}}_t(w))\mathbb{1}_{[w>w^*_t]}$.

We still have to evaluate this counterfactual for agents of age T^+ and we basically repeat the steps we have done to obtain $S^P_{T^+}$ and $S^E_{T^+}$. So we let $\dot{B}_0 := \int_0^{w^*} w\dot{m}^P_T(w)\mathrm{d}w$ and

$$\dot{B}_i := \int_{\frac{w^*}{\theta^{i-1}}}^{\frac{w^*}{\theta^i}} w\dot{m}^P_T(w)\mathrm{d}w.$$

We define $\dot{x}^S_\tau$ and $\dot{y}^S_\tau$ such that

$$\dot{x}^S_0 := \sum_{i=0}^{\infty}\dot{B}_i[\theta(1-\mu)(1-\delta)]^i,$$

$$\dot{y}^S_0 := \int_0^{w^*} w\dot{m}^E_T(w)\mathrm{d}w + \mu\sum_{i=1}^{\infty}\dot{B}_i\theta^i(1-\mu)^{i-1}(1-\delta)^i,$$

and

$$\begin{bmatrix} \dot{x}^S_{\tau+1} \\ \dot{y}^S_{\tau+1} \end{bmatrix} = \begin{bmatrix} (1-\delta)(1-\mu) & (1-\delta)\overline{p} \\ (1-\delta)\mu & (1-\delta)(1-\overline{p}) \end{bmatrix} \begin{bmatrix} \dot{x}^S_{\tau} \\ \dot{y}^S_{\tau} \end{bmatrix}.$$

Solving this difference equation we have

$$\sum_{\tau=0}^{\infty} \dot{y}^S_\tau = \frac{(\dot{x}^S_0 + \dot{y}^S_0)\mu}{\delta(\mu+\overline{p})} - \frac{\mu \dot{x}^S_0 - \overline{p}\dot{y}^S_0}{[1-(1-\delta)(1-\mu-\overline{p})](\mu+\overline{p})} =: \dot{S}^E_{T^+}.$$

Defining

$$\dot{S}^{\tilde{E}}_{T^+} := \frac{1}{\delta}\int_{w^*}^{\infty} w\dot{m}^{\tilde{E}}_T(w)\mathrm{d}w + (1-\delta)\mu \sum_{i=1}^{\infty} \dot{B}_{i+1}\frac{1-((1-\delta)\mu)^i}{1-(1-\delta)\mu},$$

we obtain

$$L^{\text{work}} = \dot{S}^E_{T^+} + \dot{S}^{\tilde{E}}_{T^+} - (S^E_{T^+} + S^{\tilde{E}}_{T^+}) + \sum_{t=0}^{T-1}\left(\int_0^{\infty} w(\dot{m}^{\tilde{E}}_t(w) + \dot{m}^E_t(w) - (m^{\tilde{E}}_t(w) + m^E_t(w)))\mathrm{d}w\right).$$

D Details on Data and Parameters' choice

Productivity

The CPS sample used to estimate the gain of productivity for each year in freedom is taken from the 2004 CPS March Supplement. We have run the regression of the logarithm of the income on years of education, age, race and gender. We exclude individuals with missing information on any this four characteristics, individuals under 18 years old and over 65 years old. The regressor age is the age of the individual minus 18. For years of education, we used a variable already built in the CPS in order to proxy the actual education. When we use the result of the population as a whole for criminals, there are two implicit hypotheses. First, the prisonal population is small enough so the depreciation for having served prison will not impact significantly in the result of this regression. Second, the productivity gain of dishonest agents when they spend one year in freedom is, in average, the same as the one of an honest agent at the same age.

Initial Distribution of incomes

The study "Survey of Inmates in State and Federal Correctional Facilities 2004" samples individuals incarcerated in 2004. This survey associates, to each individual, an interval into which the earnings of the month previous to the most recent incarceration falls. We exclude individuals who declared that part of these earnings was obtained from illegal activities. In the same way, we can obtain for each prisoner an interval into which the age falls. We suppose that agents between 18 and 24 years old have approximately the distribution of agents at 18 years old, so we exclude individuals older than 24 years old. We suppose that the initial distribution of incomes is lognormal with parameters (μ_w, σ_w). Then, using Table 8, (μ_w, σ_w) can be estimated by maximum likelihood. We have also estimated these parameters using the NLSY97, in individuals between 18 and 24 years old that have ever committed crime after 18. We have used the strong assumption that the income distribution of unemployed missing individuals is the same of the employed. Rounding to 2 decimal places, the estimation of μ_w is identical in the two methods and the estimation of σ_w increases from less than 5% comparing to the first estimation.

Year income (U$)	%	Year income (U$)	%	Year income (U$)	%
0-2400	4.97	9600-12000	9.52	24000-30000	7.13
2400-4800	10.26	12000-14400	11.06	30000-60000	8.60
4800-7200	14.73	14400-18000	12.41	60000-90000	2.15
7200-9600	7.32	18000-24000	9.22	90000-∞	2.63

Table 8: Distribution of incomes of incarcerated individuals younger than 24 years old. Year income equals 12 times the self-declared monthly earning prior to incarceration.

Total number and distribution of prisoners

We obtain the number of prisoners convicted for each type of crime in state and federal prisons from the BJS Bulletin "Prisoners in 2004". The BJS Special Report "Profile of Jail Inmates, 2002" gives us the distribution of convicted and unconvicted prisoners in jail by most serious offense in 2002. We assume the same distribution for 2004. We consider that unconvicted prisoners are exclusively in jail and we assume that the number of prisoners for larcenies of values lower than 50 dollars is negligible. We take the total number of convicted and unconvicted prisoners in jail in 2004 from the BJS Bulletin "Prison and Jail Inmates Midyear 2005". Table 9 presents the number of prisoners by type of incarceration and most serious offense.

Most serious offense	Type of incarceration			
	State prison	Federal Prison	Jail convicted	Jail unconvicted
Burglary	131200	200	18300	29100
Larceny	48100	0	21700	22700
Motor Theft	18600	0	5700	6900
Robbery	170900	8400	11100	37300

Table 9: Distribution of prisoners by type of incarceration and most serious offense.

Among the unconvicted prisoners, in the stationary state, we are interested in the ones who ended up actually convicted. This will be the case for a fraction η of unconvicted prisoners, so the total number of prisoners is given by $434200 + 96000\eta \in [434200, 530200]$. We choose $\eta = 0.5$ and we obtain 482.3 thousand prisoners in 2004.

Average time served

Data about time served is taken from the National Correction Reporting Program, between 2001 and 2013[26]. This dataset reports, at the year of the release of each prisoner convicted for a property crime in state or federal prisons, the total time served as described in Table 10.

We still need to include prisoners in jail to obtain the complete distribution of time served for all prisoners. We use three main hypotheses to do so. First, all prisoners convicted to stay in jail spend at most one year incarcerated. Second, once an unconvicted prisoner in jail is convicted, the probability of serving the sentence in jail is 13%, the proportion of number of prisoners convicted in jail over the number of convicted prisoners. Third, an unconvicted individual sentenced to serve in state or federal prison serves a time in prison according to

[26] For each year, there were at least 38 states reporting these counts.

Time served	Proportion (%)
$<$ 1 year	56.84
1 - 1.9 years	19.45
2 - 4.9 years	16.24
5 - 9.9 years	5.19
$\geq$ 10 years	2.28

Table 10: Time served, releases from state and federal prisons, 2001-2013

Table 10. We construct Table 11 to include all jail inmates in the distribution of time served under these hypotheses.

Time served	Proportion (%)
$<$ 1 year	62.49
1 - 1.9 years	16.91
2 - 4.9 years	14.41
5 - 9.9 years	4.51
$\geq$ 10 years	2.08

Table 11: Time served, releases from all correction facilities

We estimate the average sentence length by maximum likelihood supposing it is distributed as an exponential, the closest distribution to the geometric one used in the model. The parameter of this exponential distribution is 1.474 years, or 17.7 months, resulting in $\widehat{\mu}_{2004} = 0.404$.

The average property crime

We use the report Criminal Victimization of 2004 to calculate an average property crime. We consider that each victimization is the act of exactly one criminal over exactly one victim. Using also the article of Cohen (2000) with prices of 2004, we build Table 12. The average property loss and other losses is weighted with the frequency. Recall that the losses calculated for Cohen are not ex ante, so 1545 dollars is a lower bound for the average total loss inflicted on the victim by a criminal action.

Type of crime	Frequency	Losses		
	(%)	Property	Other	Total
Robbery	3.4	975	9425	10400
Burglary	22.9	1261	559	1820
Motor Theft	6.8	4290	650	4940
Theft	66.9	483	183	566
Average		931	614	1545

Table 12: The average property crime.

E Definition of the marginal effects

Consider the safety policies (k_1, μ_1), (k_0, μ_0), and $(k_s, \mu_s) = (k_0, \mu_0) + s(k_1, \mu_1)$ with crime rates equilibria given by v_1, v_0 and v_s. Define $p^0 = p(v_0, k_0)$, $p^{b,s} = p(v_0, k_s)$ and $p^{\ell,s} = p(v_s, k_0)$.

Deterrence effect In general, whenever a function of v, k or μ in Appendix B appears with a superscript b, s, it means that v is replaced by v_0 and k is replaced by k_s (for example, $\overline{\Pi}_t^{b,s} = (z(1 - p^{b,s}) - d_t^C)(1 - e^{-p^{b,s})\nu})/p^{b,s})$. The marginal deterrence effect at (k_0, μ_0) in the direction $(k_1, \mu_1) - (k_0, \mu_0)$ is defined as

$$D^{\text{det}}((k_1, \mu_1), (k_0, \mu_0)) = \lim_{s \to 0} \frac{1}{s} \left(\frac{1 - e^{-\nu p^0}}{p^0} \sum_{t=0}^{\infty} (M_t^E(w_t^{*b,s}(v_0), v_0) - N_t^E(v_0)) \right).$$

Incapacitation effect Let $\bar{\Delta}(\tau, t) := \prod_{t'=\tau}^{\tau+t-1}(1 - \delta_{t'})$ and $\Gamma(\tau, t) := \prod_{t'=\tau}^{\tau+t-1} \gamma_{t'}$ whenever $t \geq t + 1$. The marginal incapacitation effect at (k_0, μ_0) in the direction $(k_1, \mu_1) - (k_0, \mu_0)$ is defined as

$$D^{\text{inc}}((k_1, \mu_1), (k_0, \mu_0)) = \lim_{s \to 0} \frac{1}{s} \left(\sum_{\tau=0}^{\infty} M_\tau^E(w_\tau^*) \left[\frac{1 - e^{-\nu p^{b,s}}}{p^{b,s}} - \frac{1 - e^{-\nu p^0}}{p^0} \right] + \frac{1 - e^{-\nu p^0}}{p^0} \right.$$
$$\left. \times \sum_{\tau=0}^{\infty} \sum_{t=\tau+1}^{\infty} \bar{\Delta}(\tau, t) M_\tau(w_t^*/\Gamma(\tau, t)) \left((1 - e^{-\nu p^{b,s}})(1 - \mu_s)^{t-\tau-1} - (1 - e^{-\nu p^0})(1 - \mu_0)^{t-\tau-1} \right) \right).$$

Load effect Whenever a function of v, k or μ in Appendix B appears with a superscript ℓ, s, it means that v is replaced by v_s and k is replaced by k_0 (analogously with the deterrence effect, for example, $\overline{\Pi}_t^{\ell,s} = (z(1 - p^{\ell,s}) - d_t^C)(1 - e^{-p^{\ell,s})\nu})/p^{\ell,s})$. Then, the belief component of the marginal load effect at (k_0, μ_0) in the direction $(k_1, \mu_1) - (k_0, \mu_0)$ is defined as

$$D^{\text{load}}_{\text{bel}}((k_1, \mu_1), (k_0, \mu_0)) = \lim_{s \to 0} \frac{1}{s} \left(\frac{1 - e^{-\nu p^{\ell,s}}}{p^{\ell,s}} \sum_{t=0}^{\infty} (M_t^E(w_t^{*\ell,s}(v_0), v_0) - N_t^E(v_0)) \right).$$

The mechanical component of the marginal load effect at (k_0, μ_0) in the direction $(k_1, \mu_1) - (k_0, \mu_0)$ is defined as

$$D^{\text{load}}_{\text{mech}}((k_1, \mu_1), (k_0, \mu_0)) = \lim_{s \to 0} \frac{1}{s} \left(\sum_{\tau=0}^{\infty} M_\tau^E(w_\tau^*) \left[\frac{1 - e^{-\nu p^{\ell,s}}}{p^{\ell,s}} - \frac{1 - e^{-\nu p^0}}{p^0} \right] + \frac{1 - e^{-\nu p^0}}{p^0} \right.$$
$$\left. \times \sum_{\tau=0}^{\infty} \sum_{t=\tau+1}^{\infty} \bar{\Delta}(\tau, t) M_\tau(w_t^*/\Gamma(\tau, t)) \left((1 - e^{-\nu p^{\ell,s}})(1 - \mu_s)^{t-\tau-1} - (1 - e^{-\nu p^0})(1 - \mu_0)^{t-\tau-1} \right) \right).$$

The marginal load effect at (k_0, μ_0) in the direction $(k_1, \mu_1) - (k_0, \mu_0)$ is defined as

$$D^{\text{load}}((k_1, \mu_1), (k_0, \mu_0)) = D^{\text{load}}_{\text{bel}}((k_1, \mu_1), (k_0, \mu_0)) + D^{\text{load}}_{\text{mech}}((k_1, \mu_1), (k_0, \mu_0)).$$

Rehabilitation effect The total marginal effect at (k_0, μ_0) in the direction $(k_1, \mu_1) - (k_0, \mu_0)$ is given by

$$D^{\text{total}}((k_1, \mu_1), (k_0, \mu_0)) = \lim_{s \to 0} \frac{1}{s} (v_s - v_0).$$

It follows that the marginal rehabilitation effect at (k_0, μ_0) in the direction $(k_1, \mu_1) - (k_0, \mu_0)$ is given by

$$D^{\text{rehab}}((k_1, \mu_1), (k_0, \mu_0)) = D^{\text{total}}((k_1, \mu_1), (k_0, \mu_0)) - D^{\text{det}}((k_1, \mu_1), (k_0, \mu_0))$$
$$- D^{\text{inc}}((k_1, \mu_1), (k_0, \mu_0)) - D^{\text{load}}((k_1, \mu_1), (k_0, \mu_0)).$$