Inflation target expectations, transparency and monetary policy*

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Abstract

Does transparency increase the effectiveness of monetary policy? I study this question using a New Keynesian model in which firms do not observe the time-varying inflation target or monetary policy shocks. Two informational assumptions are considered: (i) firms observe the interest rate decisions only (standard assumption) and (ii) firms observe the interest rate and an idiosyncratic signal about the inflation target.

Under the standard assumption, agents infer output and inflation fluctuations by realizing that other agents are acting exactly like them. That ceases to be true when agents face strategic uncertainty induced by the idiosyncratic signal. Since inflation and output are primary determinants of the interest rate, this difference in information sets changes firms' ability to extract information from the interest rate. One key implication is that, in the case of monetary contraction, greater transparency improves the inflation-output trade-off only under the second assumption.

KEYWORDS: Transparency, inflation target, higher-order expectations, effectiveness of monetary policy, signal extraction.

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1 Introduction

Over the last few decades, one of the most remarkable changes in monetary policy has been the increase in transparency and accountability. Central banks have broadly increased their communi-

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cation about individual policy decisions by utilizing minutes and statements, in addition to more structured analysis. Another crucial aspect is the emphasis on the adoption of an explicit numerical target for inflation by several countries. These policy changes are based on the widespread belief among central banks that transparency enhances the effectiveness of monetary policy. For instance, in January 2012, the Federal Reserve decided to clarify that an inflation rate of two percent is consistent with its price stability goal in their dual mandate. The reasoning for this decision was made clear: "Such clarity facilitates well-informed decision-making by households and businesses, reduces economic and financial uncertainty, increases the effectiveness of monetary policy, and enhances transparency and accountability, which are essential in a democratic society"(FOMC; 2012).

This paper evaluates the claim that transparency increases the effectiveness of monetary policy by studying the effects of transparency about the goal of monetary policy on the trade-off between inflation and output. To address this question, I use the standard New Keynesian model in which central bank follows a Taylor rule with a time-varying, persistent, exogenous inflation target. This inflation target represents the goal of the central bank, which is not known by the private sector. I assume that firms do not observe changes in the target or monetary shocks and compare two informational assumptions: i) firms extract information about them from the interest rate, which I call imperfect common information (ICI), and ii) firms observe interest rate decisions and an idiosyncratic signal about the inflation target, which I call imperfect common knowledge (ICK), following the literature that has employed such an informational structure (see, for instance, Woodford; 2002; Adam; 2007; Nimark; 2008). Central bank communication is represented by noisy public signal about the inflation target such that its precision is interpreted as the degree of transparency (see Faust and Svensson; 2001, 2002).

This paper has two main contributions. First, I show that introducing a noisy idiosyncratic signal affects not only the weight given to both the interest rate and the private signal but also how agents extract information from interest rate decisions. Second, transparency has opposite effects on the effectiveness of the monetary policy depending on the information available to price setters (common or dispersed) and on which policy is evaluated (disinflation or usual monetary contraction). Specifically, for a disinflation policy (decrease in the inflation target), greater transparency implies a better trade-off between inflation and output under both informational assumptions. However, for monetary contraction (a positive monetary shock), improved transparency entails a better trade-off only under the dispersed information model because, in the ICK model, firms have a qualitative different learning process about the inflation target.

In the model with imperfect but common information, it is common knowledge that all firms act in exactly the same manner (share the same information and solve the same problem). This allows them to observe the price level and aggregate output. Then, since they know the monetary rule, information is extracted from the interest rate adjusted by the endogenous response to those variables. This learning process is a dynamic version of the standard signal extraction problem in which firms are unable to distinguish changes in the inflation target from monetary shocks. Thus, when firms observe a higher (adjusted) interest rate, their expectation is that it is a result of either a positive monetary shock or a lower inflation target.

The signal extraction under the dispersed information model is more complex. The idiosyncratic signal induces strategic uncertainty, i.e., firms do not know other firms' decisions in equilibrium. This prevents them from knowing current inflation and output once they are incapable of inferring the aggregate decisions from their own. Thus, strategic uncertainty affects the ability to extract information from the interest rate. In other words, when forming expectations about shocks from the observed interest rate, which obeys a Taylor rule, firms also have to consider the endogenous effects of the inflation target and monetary shocks (and their higher-order expectations) on the interest rate through the responses of inflation and output (which are known in the imperfect information case). This additional uncertainty serves as a confounding factor in the firms' signal extraction based on the Taylor rule.

The ICK model implies in a more realistic learning process. First, since firms have different information sets, they disagree in their inflation expectations both regarding short and long-term expectations, which is a common feature in inflation expectations data. Second, the learning process is intrinsically related to the dynamics of the model. The uncertainty regarding inflation and output translates into an additional factor that agents have to take into account when trying to understand the reasons why the interest rate have changed. Under imperfect information, the signal extraction depends only on the characteristics of the shocks and not on how they affect the economy.

As an example, suppose that firms observe an increase in the interest rate owing to a monetary shock. They have to consider not only the direct effect of shocks (as in the ICI model) but also whether this increase is a response to higher inflation and/or output, which, in turn, could be a response to a negative monetary shock or a higher target. Therefore, the signal extraction problem must balance the endogenous and direct effects of the shocks on interest rate, which are affected by shocks in opposite directions. Since the inflation target process is very persistent, a change in the target has strong effects on endogenous variables. The opposite is true for monetary shocks since they are temporary. Hence, the endogenous effect is dominant for the former and dominated for the latter. This implies that if firms observe a higher interest rate after a positive monetary shock, they expect a higher inflation target – the opposite situation as in the ICI model.

Learning differences about the inflation target play an essential role in the inflation-output trade-off. Intuitively, since target shocks are very persistent, they have strong effects on inflation expectations, implying sizable shifts in the Phillips curve. Differences in learning about monetary shocks have a small role since they entail smaller shifts owing to their temporary nature. Interestingly, in the case of a monetary shock in the ICK model, since firms expect a increase in the target, there is a worse trade-off between inflation and output than in the full information model. The opposite is true in the ICI model: firms expect a lower target which implies a better trade-off. For the case of the target shock, the differences among the ICK and ICI models, regarding expectations about the monetary shock, are not sufficient to change the trade-off. Specifically, both models have a worse inflation-output trade-off than the full information model.

Building on the contributions of Nimark (2008) and Melosi (2017), I propose a solution method for the imperfect common knowledge model that includes exogenous public signals into agents' information sets. Within this framework, a lower standard deviation of the public signal about the target represents a higher degree of transparency of the central bank. Therefore, by providing a more accurate public signal about the target, greater transparency affects the economy through agents' expectations. Firms can better assess whether the change in the interest rate was a response to monetary shocks or a change in the inflation target.

Greater transparency implies that both models are closer to the full information model, but it does not change the qualitative differences in expectations formation. Therefore, transparency about the target implies lower disinflation costs, independent of the assumptions considered because both models have a worse inflation-output trade-off than the full information counterpart. For monetary shocks, however, since the trade-off in the ICK model is worse than in the full information model, higher transparency implies a better trade-off. Precisely the opposite happens in the ICI model since it entails a better trade-off between inflation and output for this particular shock.

This paper is organized as follows. Section 2 discuss the literature regarding changes in monetary policy rules, central bank transparency and imperfect common knowledge. Section 3 presents the models and discusses their differences in the signal extraction problem due to the informational assumptions. Then, sections 4 and 5 relate the signal extraction differences to the trade-off between inflation and output and the effects of transparency, respectively. Section 6 concludes the paper.

2 Related literature

This section presents a brief review of the related literature. In subsection 2.1, I relate the model to the literature that studies shifts in monetary policy rules. Then, subsection 2.2 relates the model to the central bank transparency literature. Finally, in subsection 2.3, I discuss the literature regarding the imperfect common knowledge model.

2.1 Changes in monetary policy rules

Several papers study changes in monetary policy rules and their impact on inflation dynamics and the overall economy. Clarida et al. (2000) and Lubik and Schorfheide (2004) present evidence that US monetary policy authority has increased the interest rate response to inflation since the beginning of the 1980s. Both argue that before this change, monetary policy was not sufficiently reactive to inflation fluctuations and thus the central bank was unable to stabilize inflation. Therefore, the change in monetary policy is the main reason for the inflation stabilization in the 1980s.

Two main aspects of this interpretation have been questioned. First, the influential paper from Sims and Zha (2006) shows that changes in monetary policy are mainly stochastic (and not a deterministic one-time change) and that regime changes in the volatility of shocks are more important for explaining the stabilization of inflation. Second, Cogley et al. (2010) do not see the parameters of the Taylor rule as the determinant of the stabilization of inflation but instead consider changes in the inflation target to be the key explanation. Moreover, Schorfheide (2005) assumes that changes in monetary policy can be described as a Markov-switching regime in the inflation target and considers the cases in which the current regime is known and that in which it is not; in the latter case, agents use Bayesian updating to infer the monetary policy regime.

Ireland (2007) discusses the sources of such changes by estimating a DSGE model that allows the inflation target to be a response to structural temporary supply shocks in addition to exogenous changes. He finds evidence that during the high-inflation periods of the 1970s, transitory supply shocks had persistent effects on inflation owing to unwillingness of the Federal Reserve to incur output costs.

More related to this study, Erceg and Levin (2003) and Andolfatto et al. (2008) work with a model in which agents cannot disentangle persistent shifts in the inflation target from transitory shocks to the monetary policy rule. Thus, agents form rational expectations given their observations of interest rate decisions. In this case, inflation has an inertial behavior as a result of the persistence of agents' optimal forecast of unobserved quantities. This has important implications for the costs of disinflation and the existence of persistence in the inflation forecast errors. Therefore, inflation persistence is not an intrinsic feature of the economy but rather depends on the perception about the current monetary stance, i.e., the expectations about the inflation target and monetary shocks.[1] I show that central bank communication about the target can diminish the uncertainty about the target, implying a more accurate and faster learning process about the target and thus leading to lower output costs since the inflation expectations change accordingly.

[1] Inflation persistence can also depend on expectations regarding other shocks that affect inflation but are not related to monetary policy.

2.2 Central bank transparency

Since the increases in transparency by central banks, the topic has drawn a lot of attention, particularly in the empirical literature.[2] Here, I will briefly discuss the theoretical literature. Since most economic decisions are forward-looking, communication and transparency are essential tools for central banks (see, for instance, Woodford; 2001, 2005). More information for agents can diminish uncertainty, thereby leading to more accurate and better decisions.

Most of the theoretical literature regarding central bank transparency builds on the Barro-Gordon framework, which focuses on the inflation bias and reputation issues. Geraats (2002) surveys the main results of this literature in which transparency (opacity) is usually defined as symmetric (asymmetric) information about the issue in question, e.g., the inflation target. In this literature, typically, the central bank is either fully transparent – e.g., it announces the inflation target – or fully opaque. Faust and Svensson (2001, 2002) are exceptions that allow a degree of transparency depending on the precision of the noise in the public signal announced by the central bank. This paper follows this approach by introducing a public signal about the inflation target.[3]

Geraats (2002) distinguishes two effects of transparency. First, asymmetric information implies higher uncertainty for agents that have an informational disadvantage, called "uncertainty effects". Second, there exists an "incentive effect", which is the central bank's willingness to manipulate the beliefs of others through signaling. Transparency will influence both effects, diminishing (vanishing) the uncertainty effect, which is welfare-improving, and mitigating the incentive effect, which may or may not be welfare-improving.[4]

More related to the modeling approach used in this paper, Morris and Shin (2002) show that agents whose actions are strategic complements and decided based on both public and private information put a disproportionately high weight on public information. Thus, sufficiently imprecise public information could generate undesired volatility in agents' decisions, which might be welfare-reducing. Svensson (2006) argues that in practice, this result is pro-transparency since public information is likely to be more accurate than private information. As in Morris and Shin (2002), the solution method proposed in this paper allows agents to observe both private and public signals, but here agents learn about dynamic fundamentals.

Another argument against central bank transparency from Morris and Shin (2005) is that higher transparency implies less informationally efficient markets. The idea is that when more public information about the economy is available, market prices and interest rates – which depend on expectations about the state of the economy – will respond less to the actual state of the econ-

[2] Blinder et al. (2008) presents a comprehensive survey of the empirical findings.

[3] In Faust and Svensson (2001, 2002), the public signal is about the control error of inflation.

[4] Geraats (2002) shows an example in which a central bank being transparent about the target eliminates uncertainty about inflation and output but also removes the reputation considerations that reduce the inflation bias. It turns out that the latter effect dominates and thus opacity is desirable in this example.

omy but rather more strongly reflect information about the public information itself. Therefore, equilibrium market outcomes provide less information about economic fundamentals.

This paper focuses on the transparency about the inflation target within the New Keynesian framework, which now is the benchmark model for monetary policy evaluations. Eusepi and Preston (2010) and Ascari et al. (2017) also use the New Keynesian model to study the implications of transparency, but they emphasize the effects on the expectations stability properties of the model.[5] Moreover, while the former considers transparency both about the policy rule and the inflation target, the latter considers only transparency about the rule. Here the policy rule is common knowledge, but agents do not observe its shocks.

2.3 Imperfect common knowledge

This literature introduces an idiosyncratic signal about an unobserved fundamental. This framework contains the usual perfect information model (common knowledge of information) as a particular case when the variance of the idiosyncratic signal goes to zero. Therefore, this imperfect and dispersed noisy information structure is usually referred to as imperfect common knowledge (ICK) (see, for instance Woodford; 2002; Adam; 2007; Nimark; 2008).

A growing body of literature uses this information setup to explain business cycles (see, e.g., Lorenzoni; 2009; Angeletos and La'O; 2010, 2013), to understand the real effects of monetary policy (Woodford; 2002) and to study optimal monetary policy (see Lorenzoni; 2010; Adam; 2007 and Paciello and Wiederholt; 2013 for models that include rational inattention). Angeletos and La'O (2011) also study optimal monetary policy under nominal rigidities within a more general setup for informational frictions.

Closer to this paper, Nimark (2008) studies a model which the main feature is imperfect common knowledge about the aggregate real marginal cost with sticky prices. Specifically, firms do not observe other firms' marginal cost. Since there are strategic complementarities in pricing decisions, firms must forecast other firms' pricing decisions to make their own decisions.

I propose a model with ICK about the inflation target such that agents observe interest rate decisions and their signal to predict other firms' forecast. Melosi (2017) studies the signaling channel of monetary policy in the same manner, but in his model, agents learn about supply, demand and, monetary shocks. Instead, I consider that firms extract from the interest rate information about changes in the inflation target and monetary shocks.

This paper compares the signal extraction problem under the dispersed and noisy information

[5] Those papers use the expectations stability concept from Evans and Honkapohja (2001) that evaluates whether the learning process converges to the one implied under full information rational expectations. In that literature, agents do not know some parameters of the model. They behave as econometricians and compute expectations according to a reduced-form model. In the learning process applied in this paper, agents have rational but limited information. The learning process is about unknown stochastic processes instead of parameters.

model proposed with the one from the imperfect information model. The latter is a fully forward-looking version of the models of Andolfatto et al. (2008), Erceg and Levin (2003) and Del Negro and Eusepi (2011). I show that since agents are extracting information from the interest rate, the learning processes under those assumptions are qualitatively different. Those differences have important implications on the effects of transparency of the central bank on the inflation-output trade-off.

I also contribute to the literature by proposing a solution method that includes exogenous public signals into agents' information set. This solution method builds on the contributions of Nimark (2008) and Melosi (2017). The solution is a dynamic version of the static model with idiosyncratic and public signals from Morris and Shin (2002).

3 Model

Apart from the introduction of an imperfectly observed inflation target, the framework in the following is a standard New Keynesian model with sticky prices and monopolistic competition. The economy is populated by a representative household, a continuum of monopolistic competitive firms and a monetary authority. The central bank chooses the interest rate following a Taylor rule with a time-varying target that is not observed by the private sector. Firms choose their price taking into account the probability of not being able to readjust prices subject to their information. I consider two different informational assumptions: i) imperfect common information, where firms observe the interest rate chosen by the central bank and form expectations about the target and the monetary shock based only on this information, and ii) imperfect (common) knowledge, where agents observe both the interest rate and an idiosyncratic signal about the inflation target. For simplicity, perfect information for households is assumed.[6]

The model has the following timing protocol. Every period t is divided into two stages, in which actions are taken simultaneously. In stage 1, the inflation target and monetary shocks realize, the central bank sets its interest rate and, in the ICK model, firms observe their idiosyncratic signal s_{it}. In stage 2, given the observables, firms set their price $P_{i,t}$ and hire labor $L_{i,t}$ to produce and deliver the demanded quantity, $C_{i,t}$. Households also decide their consumption composite C_t, labor supply L_t and demand for one-period nominal bonds, B_t, and markets clear.

3.1 Households

There is a representative household whose utility function is given by

[6]This assumption do not affect the results qualitatively.

$$U = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{C_\tau^{1-\gamma}}{1-\gamma} - \frac{L_\tau^{1+\varphi}}{1+\varphi} \right), \tag{1}$$

where $0 < \beta < 1$ is the discount factor, γ is the inverse of the intertemporal rate of substitution, and φ is the inverse of the Frisch labor supply elasticity. For simplicity, I assume that households have full information with expectation operator denoted by $\mathbb{E}_t$.

The composite consumption good is a Dixit-Stiglitz aggregator $C_t = \left(\int_0^1 (C_{i,t})^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}$, where $C_{i,t}$ is the consumption of each variety i in period t and $\varepsilon > 1$ is the elasticity of substitution between varieties. The household budget constraint is given by

$$P_t C_t + B_t = W_t L_t + T_t + (1 + I_{t-1}) B_{t-1}. \tag{2}$$

where B_t is the quantity of one-period nominal bond that the household holds in period t that pays the interest rate I_t. L_t denotes the hours of labor of the representative household that receives the hourly nominal wage, W_t, and T_t is the nominal net transferences.

The household problem can be divided into two parts. First, she chooses her optimal allocation of consumption between varieties by maximizing the consumption bundle, C_t, subject to the consumption budget. Therefore, the demand for each good i is given by

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} C_t, \tag{3}$$

where the price level of the composite good is given by $P_t = \left(\int_0^1 (P_{i,t})^{(1-\varepsilon)} di \right)^{1/(1-\varepsilon)}$. Second, the household chooses her optimal allocation between bonds, labor, and consumption that maximizes the utility function subject to the household budget constraint.

3.2 Monetary authority

The monetary authority sets the interest rate according to a log-linearized Taylor-rule type of reaction function given by

$$i_t = \pi_t^* + \phi_\pi (\pi_t - \pi_t^*) + \phi_y y_t + \eta_t, \tag{4}$$

where π_t is the (gross) inflation, y_t denotes the output, and η_t is a monetary shock. Henceforth, all lowercase variables refer to the log-deviation of the respective uppercase variables from their respective steady-state values. Therefore, $\pi_t^* = log(\Pi_t^*) - log(\Pi^*)$ is the log-deviation of the inflation target from its steady-state value, Π^*, which is constant and common knowledge.

To avoid indeterminacy issues, the response to inflation is chosen such that $\phi_\pi > 1$. Furthermore, I assume that the log-linearized inflation target and monetary shock follow autoregressive processes:

$$\pi^*_t = \rho_\pi \pi^*_{t-1} + \varepsilon^\pi_t \tag{5}$$

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon^\eta_t \tag{6}$$

where $\varepsilon^j_t \sim \mathcal{N}(0, \sigma^2_j)$, for $j = \{\eta, \pi\}$.

Note that the assumed reaction function and the inclusion of an inflation target do not necessarily mean that the model applies only to inflation targeting regimes. It describes any central bank that seeks to offset the developments of inflation and output by increasing the interest rate based on an implicit target that can vary over time. As discussed in the literature review, there is no definitive method for modeling changes in the monetary policy rule. I follow the recent literature that models changes in monetary policy as a monetary rule with a time-varying exogenous inflation target (see Erceg and Levin; 2003; Ireland; 2007; Andolfatto et al.; 2008; Cogley et al.; 2010; Del Negro and Eusepi; 2011).

One alternative to the proposed approach is to let the Taylor rule coefficients (ϕ_π and/or ϕ_y) vary over time and adopt a fixed inflation target. However, as pointed out by Schorfheide (2005), changes in the inflation target are important up to a first-order approximation, whereas changes in the response of the interest rate to deviations of inflation from the inflation target (and/or output) are a second-order effect.[7] Moreover, there is evidence that inflation has a very smooth trend whose variation explains most of the unconditional variance of inflation (see the evidence for the US presented in Stock and Watson (2007) and Cogley et al. (2010); see Garnier et al. (2015) for 14 advanced economies). Assuming a very persistent process for the inflation target is a straightforward method to incorporate this behavior of the trend inflation.

Previous literature, e.g., Ball (1994a) and Mankiw and Reis (2002) define a disinflation episode as permanent changes in money supply growth. Since the monetary authority is setting the interest rate, disinflation is a permanent drop in the inflation target. I follow Erceg and Levin (2003) by modeling the target as a very persistent but still stationary process.

3.3 Pricing decisions and information

There is a continuum $i \in [0, 1]$ of monopolistic competition firms that produce a differentiated good Y_{it} using a linear production function with labor as its only input such that

$$Y_{it} = L^\alpha_{it}, \tag{7}$$

[7] This is not inconsistent with the arguments of Clarida et al. (2000) and Lubik and Schorfheide (2004), since those authors argue for a one-time permanent change in the coefficients of the Taylor rule. Temporary changes in those coefficients are a second-order effect.

where $\alpha \in (0, 1]$. Introducing decreasing returns into the production function increases the so called "real rigidities" that help pricing decisions to be strategic complements. As in Calvo (1983), in each period, there is a constant probability $(1-\theta)$ that each firm will re-optimize its price. Those firms that do not optimize their prices are assumed to adjust mechanically their prices by the steady-state inflation, Π^*.[8] This assumption implies that the evolution of the price index is given by

$$P_t = \left[\theta(\Pi^* P_{t-1})^{1-\varepsilon} + (1-\theta)\int_0^1 (P_{it}^*)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}, \tag{8}$$

where P_{it}^* denotes the optimal price that firm i is able to choose. Firms maximize the expected discounted value of their profit, which is given by

$$E_{i,t}\left[\sum_{s=0}^{\infty}(\beta\theta)^s \Lambda_{t,t+s}\left((\Pi^*)^s P_{it}^* - P_{t+s} MC_{i,t+s}\right) Y_{i,t+s}\right], \tag{9}$$

subject to the market clearing condition, $Y_{i,t} = C_{i,t}$, the firm individual firm demand (3) and the production function (7). $\Lambda_{t,t+s}$ is the stochastic discount factor, $MC_{i,t+s}$ is the real marginal cost, and $E_{it}[\cdot] \equiv E[\cdot|\mathcal{I}_t^i]$ is the expectation operator of firm i conditional on the current information set, $\mathcal{I}_t^i$.

The full information model is compared with two type of informational assumptions. In the ICI model, firms observe only the interest rate decision. Therefore, they will extract information from the Taylor rule (4) to form expectations about the inflation target and the monetary shock. Meanwhile, in the ICK model, firms observe, in addition to the interest rate, an idiosyncratic signal about the inflation target given by

$$s_{it} = \pi_t^* + v_{it}, \tag{10}$$

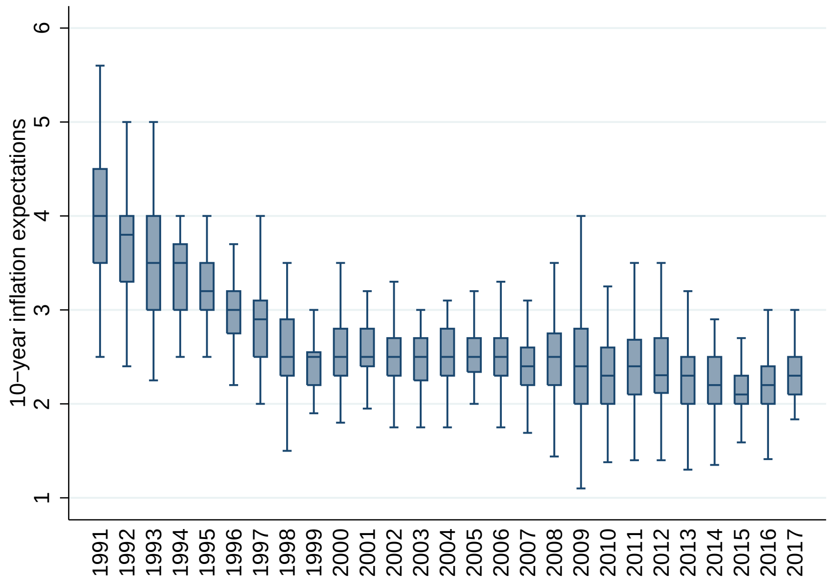
where $v_{it} \sim \mathcal{N}(0, \sigma_v^2)$. In this context, the interest rate works as an (endogenous) public signal since all firms share this observation.

Firms observe their idiosyncratic signal and do not communicate among themselves. This implies that there is disagreement in expectations even in the steady-state. Figure 1 shows the cross-sectional distribution of the 10-year ahead inflation expectations (CPI for the US) from the Survey of Professional Forecasters (SPF) over time during 1991 to 2017.[9] By observing the figure,

[8] This type of indexing mechanism prevents the steady-state inflation from impacting the steady-state output level owing to a mechanical effect on the relative prices of re-optimizing firms vis-a-vis the firms not able to adjust. See Ascari (2004) for details.

[9] The Survey of Professional Forecaster data for 10-year ahead expectations data has limited availability in comparison with short-term expectations about the same price indexes. The longest data available is for the CPI inflation, which starts in Q4:1991.

Figure 1: Distribution of the 10-year ahead inflation expectations



Note: Individual data for the 10-year ahead CPI inflation from the Survey of Professional Forecasters. Data is quarterly, but the percentiles are calculated pooling observations from same year. The year of 1991 uses only data of the last quarter, and the year of 2017 uses only the first three quarters due to availability. The boxplot excludes outlier values.

two features of the data draw attention. First, the long-term expectations gradually decreased during the 1990s and became stable in 2000s and afterward. This feature can be consistent with persistent changes in the inflation target that I advocate here. Second, and more importantly, there is significant disagreement in the data. By definition, the ICI model cannot account for this since all agents share the same information. The introduction of the idiosyncratic signal allows the ICK model to be consistent with disagreement in long-term expectations presented in the data.[10]

In the following, I present the differences in the solution and expectations formation under such assumptions about the information available to firms. Then, I discuss the implications for the trade-off between inflation and output and the effects of transparency.

3.4 Log-linearized model and solution

The household's optimization solution is the standard Euler equation. Its log-linearized version is given by

[10]By the interquartile range implied in the boxplot, one can see that the dispersion changes over time. The ICK model cannot account for that feature since the disagreement in the model is constant. Mankiw et al. (2004) document similar time-varying dispersion for short-term expectations data and show that the sticky information model can account for this feature.

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\gamma}\left[i_t - \mathbb{E}_t \pi_{t+1}\right], \tag{11}$$

where the aggregate constraint that $y_t = c_t$ was used. y_t denotes the output, i_t is the one-period (gross) interest rate, and π_t is the (gross) inflation rate. The log-linearized version of the price level equation (8) is

$$p_t = \theta p_{t-1} + (1-\theta) p_t^*, \tag{12}$$

where p_{it}^* is the optimal price of firm i and $p_t^* = \int_0^1 p_{it}^* di$ is its average. The log-linearized solution of the firms' problem is

$$p_{i,t}^* = (1-\beta\theta) E_{i,t}\left[\sum_{s=0}^{\infty} (\beta\theta)^s \left(p_{t+s} + \xi y_{t+s}\right)\right], \tag{13}$$

where $\xi = \frac{\alpha(\gamma-1)+1+\varphi}{\alpha+(1-\alpha)\varepsilon}$ is a parameter that controls the strategic complementarity (or substitutability) of firms' price decisions.[11] Therefore, in order to choose their optimal prices, firms use their individual information to forecast the paths of y_t and p_t

Using equations (13) and (12) and after some algebra, one can obtain the imperfect common knowledge New Keynesian Phillips Curve (ICK-NKPC):

$$\pi_t = (1-\theta) E_t^{(1)}\left[\pi_t\right] + \beta\theta E_t^{(1)}\left[\pi_{t+1}\right] + \kappa\theta E_t^{(1)}\left[y_t\right], \tag{14}$$

where $\kappa = \xi \frac{(1-\beta\theta)(1-\theta)}{\theta}$ and $E_t^{(1)}[\cdot] = \int_0^1 E_{it}[\cdot]\, di$ is the average expectation operator. Therefore, the equilibrium responses of the endogenous variables satisfy the Euler equation (11), the Phillips curve (14) and the Taylor rule (4).

In the case in which firms have full information, i.e., $E_{i,t}[\cdot] \equiv \mathbb{E}_t[\cdot]$, equation (14) becomes the standard New Keynesian Phillips curve, $\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa y_t$. In the ICI model, the current inflation and output are known – a feature that I emphasize latter – and thus, equation (14) becomes the imperfect information Phillips curve, which is given by

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \kappa y_t, \tag{15}$$

where firms have a common expectation $E_t[\cdot] \equiv E[\cdot|\mathcal{I}_t]$ since they have the same information set, $\mathcal{I}_t$.

The solution of the imperfect information model is given by the matrices Q_1 and Q_2 such that

$$Y_t = Q_1 x_t + Q_2 E_t\left[x_t\right], \tag{16}$$

[11] See Woodford (2003) for a detailed discussion.

where $Y_t = [y_t \quad \pi_t \quad i_t]'$ is the vector of endogenous variables and $x_t = [\eta_t \quad \pi_t^*]'$ is the vector of unobserved shocks.[12]

Finally, in the ICK model, each firm uses the information from the interest rate and its own signal to choose prices. The idiosyncratic information induces strategic uncertainty, i.e., firms do not know other firms' pricing decisions since they do not share the same information. Nimark (2008) shows that under this environment, by taking higher-order average expectations and substituting them iteratively, equation (14) can be rewritten as

$$\pi_t = \kappa\theta \sum_{k=1}^{\infty} (1-\theta)^{k-1} E_t^{(k)}[y_t] + \beta\theta \sum_{k=1}^{\infty} (1-\theta)^{k-1} E_t^{(k)}[\pi_{t+1}], \tag{17}$$

where $E_t^{(k)}[.]$ denotes the average k-th order expectation defined by $E_s^{(k)}[z_t] = \int_0^1 E_{is}\left[E_s^{(k-1)}[z_t]\right] di$ for $k \geq 1$, any periods $s \leq t$ and any variable z_t. For convenience, I define $E_s^{(0)}[z_t] = z_t$ such that the first-order average expectation is as defined above. Equation (17) emphasizes the well-known result that higher-order expectations matter when there are strategic complementarities and strategic uncertainty.

Nimark (2008) proposes a method that provides an approximate solution with a finite number of orders of expectations. The solution can be arbitrarily accurate since the effect of higher-order expectations on inflation decrease as the order increases.[13] Therefore, there is a matrix Q such that the model solution is given by

$$Y_t = Q x_t^{(0:\bar{k})}, \tag{18}$$

where $x_t^{(0:\bar{k})}$ denote the expectations hierarchy, $x_t^{(0:\bar{k})} \equiv \begin{bmatrix} x_t' & E_t^{(1)}[x_t]' & \cdots & E_t^{(\bar{k})}[x_t]' \end{bmatrix}'$, the vector that stacks the average higher-order expectations from order 0 to $\bar{k}$.

Then, one can guess (and verify) that the dynamics of the expectations hierarchy can be defined as

$$x_t^{(0:\bar{k})} = A x_{t-1}^{(0:\bar{k})} + B\varepsilon_t, \tag{19}$$

where $\varepsilon_t = [\varepsilon_t^\eta \quad \varepsilon_t^\pi]'$. Following Nimark (2008), in the Appendix A.3, it is shown how the guessed matrices A, B and Q are pinned-down in equilibrium, and an algorithm to determine the fixed point implicit in the interaction of the expectations hierarchy (solution for A and B) and the response of endogenous variables to the hierarchy (solution for Q) is presented.

In the following, I show how expectations are formed in both models and highlight the differ-

[12] An straightforward method to implement the solution of the ICI model is to write it as in the standard linear system of (full information) rational expectation equations considering the expectation $E_t[x_t]$ and x_t as correlated shocks. Then, the dynamics of x_t and $E_t[x_t]$ defined in the following by equations (22) and (23), respectively. See Blanchard et al. (2013) for similar approach.

[13] In equation (17), the coefficients of the k-th order expectation for the output and next-period inflation depend on $(1-\theta)^{k-1}$, which is decreasing in k.

ences in learning and their implications.

3.5 Signal extraction and expectations formation

A vast body of literature has studied the implications of dispersed information and higher-order expectations for business cycles and monetary policy. Less attention has been paid to the implications of the imperfect common knowledge information structure for extracting information from endogenous variables. One exception is the literature the discuss welfare consequences of the public information availability. If public signals become very precise, endogenous variables such as prices and interest rates become less informative about the economic fundamentals. Those variables reflect more information about the public signal instead of fundamentals, which may be welfare decreasing (see Morris and Shin; 2005; Amador and Weill; 2010, 2012; Colombo et al.; 2014; Kohlhas; 2016).

This paper, instead, discuss the implications of the strategic uncertainty induced by the noisy private signal to the learning process from the interest rate. This section highlights differences in learning process between the ICI and ICK models.

3.5.1 ICI model: standard signal extraction

In the imperfect common information (ICI) model, agents observe the whole history of endogenous variables (prices, output and interest rate). Thus, the common information set of firms in period t is given by

$$\mathcal{I}_t = \{i_\tau, \pi_\tau, y_\tau | \tau \leq t\}. \tag{20}$$

Firms, however, do not observe all the history of the monetary and inflation target shocks. Since both shocks are within the Taylor rule, inflation and output provide information about those shocks only through their role in the monetary rule. More specifically, firms compute the "adjusted interest rate", $\hat{i}_t \equiv i_t - \phi_\pi \pi_t - \phi_y y_t$, i.e., the interest rate discounted by the endogenous responses to inflation and output. Therefore, all the information content from firms' information set is summarized by the variable $\hat{i}_t$. Then, the firms' observational equation is given by

$$\hat{i}_t = \eta_t - (\phi_\pi - 1)\pi^*_t = C_1 x_t, \tag{21}$$

where $C_1 = [1 \quad -(\phi_\pi - 1)]$ and $x_t \equiv [\eta_t \quad \pi^*_t]'$. Therefore, the expectations formation is a standard dynamic signal extraction problem: firms observe a linear combination of shocks but cannot disentangle inflation target changes from monetary shocks. This is the same approach used by Erceg and Levin (2003), Schorfheide (2005), Andolfatto et al. (2008), Del Negro and Eusepi (2011).

Since the model is linear and shocks are normally distributed, the rational expectation is given by the Kalman filter with the observational equation (21) and the state equation is given by

$$x_t = A_1 x_{t-1} + \varepsilon_t, \tag{22}$$

where $A_1 = diag(\rho_\eta \quad \rho_\pi)$ and $\varepsilon_t = (\varepsilon_t^\eta \quad \varepsilon_t^\pi)$.

The following proposition uses the Kalman filter to write down the expectations formation for the inflation target and monetary shocks.

Proposition 1. *Given the observational equation (21), the rational expectation about unobservable shocks, x_t, is given by*

$$E_t(x_t) = (I_n - \tilde{K}C_1)A_1 E_{t-1}(x_{t-1}) + \tilde{K}C_1 A_1 x_{t-1} + \tilde{K}C_1 \varepsilon_t, \tag{23}$$

where $\tilde{K} = \tilde{P}C_1' \left[C_1 \tilde{P} C_1'\right]^{-1}$ and $\tilde{P} = A_1(I_n - \tilde{K}C_1)\tilde{P}A_1' + \Sigma_\varepsilon$ are the steady-state Kalman gain matrix and the mean square error of the prediction error of the state, respectively.

The proof is presented in the Appendix A.1. The steady-state Kalman gain matrix $\tilde{K}$ provides the optimal stationary response of the expectation to changes in the observable $\hat{i}_t$. As usual, this matrix depends on the ratio σ_η/σ_π, not on the variances. In this proposition, instead of writing the expectation as a function of the observation, $\hat{i}_t$, I write it as a function of the shocks itself. This form will be helpful to compare to the expectations formation of the ICK model and to solve the model.

3.5.2 ICK model: signal extraction from private information and endogenous variables

There are two additional features in the imperfect common knowledge (ICK) model. First, firms also observe the private signal s_{it} from equation (10). Second, I assume the following timing protocol. Every period t is divided into two stages. In stage 1, the inflation target and monetary shocks, and the idiosyncratic signal realize, the central bank sets its interest rate. Firms observe i_t and s_{it}, and set their price $P_{i,t}$ given their information, and (credibly) commit to satisfy the demand at the chosen prices. In stage 2, firms hire labor $L_{i,t}$ to produce and deliver the demanded quantity, $C_{i,t}$. Households also decide their consumption composite C_t, labor supply L_t and demand for one-period nominal bonds, B_t, and markets clear.[14]

[14] In the ICI model, the assumption that agents observe the endogenous variables is equivalent to assume that firms decide prices and quantities simultaneously, which implies that firms can infer the price level and aggregate output via their own price and production decisions. If the timing were the same as in the ICK model, the learning process would be different because firms would not infer y_t by observing y_{it}. Under this alternative timing assumption, the solution method must be modified. The modified method is similar to the approach used in the ICK model described below. Theoretically, this assumption weakens the difference in expectation formation between

This timing is a standard assumption in the literature (Adam; 2007; Nimark; 2008; Angeletos and La'O; 2009, 2011). Angeletos and La'O (2011) emphasize the importance of this timing in pricing and quantity decisions in the presence of informational frictions, but only in the context of exogenous signals. Here, the timing assumption also affects the signal extraction from the interest rate.

The idiosyncratic signal introduces strategic uncertainty in pricing decisions, which has two key implications for the model. First, the optimal prices of individual firms depend on the aggregate price level. Since firms do not know other firms' pricing decisions, they have to form expectations about this price level. This implies that when aggregating to compute the price level, the price will depend on the average expectation about itself. Therefore, the first-order average expectation of the price will depend on the second-order average, and so on. Then, the ICK Phillips curve (14) depends on the hierarchy of expectations about current and future inflation and output. This is emphasized by Nimark (2008) in the case of the New Keynesian Phillips curve, but it is also true for many other environments with strategic complementarities (see Morris and Shin; 2002; Woodford; 2002; Nimark; 2008; Angeletos and La'O; 2009, 2013).

The second implication, which is key for the results, is that the strategic uncertainty affects the capabilities of agents to infer aggregate decisions from their own choices. Specifically, because firms do not known other firms' pricing decisions, they cannot infer the average optimal price $p_t^* = \int_0^1 p_{it}^* di$. Therefore, firms cannot infer the price level from equation (12), as they can in the model with imperfect information. Moreover, because firms set prices before production takes place and wages are determined, firms cannot infer the aggregate output. Therefore, the information set of firm $i \in [0,1]$ in period t is given by

$$\mathcal{I}_t^i = \{i_\tau, s_{i\tau}, \pi_{\tau-1}, y_{\tau-1} | \tau \leq t\}. \tag{24}$$

That is, firms observe the history of the interest rate and the private signal but only the past history of inflation and output. This has important implications for how firms extract information from the interest rate. To clarify this point, note that the observational equations are the private signal (10) and the Taylor rule (4). The effect of monetary and inflation target shocks on the interest rate can be decomposed into a direct effect and an endogenous effect such that

$$i_t = \underbrace{\phi_\pi \pi_t + \phi_y y_t}_{\text{Endogenous effect}} + \underbrace{\eta_t - (\phi_\pi - 1)\pi_t^*}_{\text{Direct effect}}. \tag{25}$$

In the ICI model, since firms observe π_t and y_t, the endogenous effect is known. Then, firms observe the direct effect, but there is uncertainty about its source. Thus, firms can extract infor-

the ICI and ICK models. In practice, this does not play an important role because interest rates react mostly to inflation, and not to output, i.e., ϕ_y is close to zero, and ϕ_π is greater than one.

mation from the adjusted interest rate shown in equation (21), which is by definition the direct effect. In contrast, in the ICK model, there is also uncertainty about the endogenous effect. Interestingly, this results in a learning process that is quite different from the one of the ICI model. For instance, suppose that agents observe an increase in the interest rate. For the ICI model, firms with the information set $\mathcal{I}_t$ observe an increase in $\hat{i}_t$ (positive direct effect), and thus firms expect that it was either a response to a positive monetary shock or a decrease in inflation target (recall that $\phi_\pi > 1$). For the ICK model, a firm with the information set $\mathcal{I}_t^i$ also considers the endogenous effects of inflation and output. Note, however, that higher inflation and output are likely to be responses to a *negative* monetary policy shock or an *increase* in the target. Therefore, the sign of the expectations about shocks is ambiguous, depending on the relative size of the endogenous and direct effects, because they provide information about shocks in opposite directions. Moreover, since the endogenous variables depend on the whole hierarchy of expectations, firms also have to consider the impact of the higher-order expectations of the shocks on the interest rate through their effects on inflation and output. For the same reason, the parameters may affect the expectations formation because they impact the endogenous responses of inflation and output to shocks and their expectations hierarchy, whereas the learning process of the ICI model is affected only by parameters related to the exogenous shocks.

In the following, I present how firms optimally form expectations considering both effects. By the same reasoning as before, the rational expectations can be computed using the Kalman filter, but with different state and observational equations. The state equation is the expectations hierarchy from equation (19). The observational equation will reflect the two aforementioned differences. Because π_t and y_t are not observed, firms use their knowledge of the model and the equilibrium interest rate from equation (18), and they include the private signal from equation (10). Therefore, the observational equation can be written as

$$\begin{bmatrix} i_t \\ s_{it} \end{bmatrix} = \begin{bmatrix} Q_i \\ e_{\pi^*} \end{bmatrix} x_t^{(0:\bar{k})} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_{it} = C x_t^{(0:\bar{k})} + D v_{it}, \tag{26}$$

where Q_i is the $1 \times 2(\bar{k}+1)$ subvector of matrix Q from equation (18). It describes the equilibrium response of the interest rate to the hierarchy of expectations. e_{π^*} is the selection matrix such that $\pi_t^* = e_{\pi^*} x_t^{(0:\bar{k})}$, and C and D are defined accordingly. Since the interest rate obeys the Taylor rule, the equilibrium Q_i has to be such that

$$Q_i = \phi_\pi Q_\pi + \phi_y Q_y + C_1 e_x, \tag{27}$$

where Q_π and Q_y are the analogous equilibrium responses of inflation and output to the hierarchy of expectations. C_1 is defined in the same manner as in the observation equation of the ICI model (see equation (21)), and e_x is the selection matrix such that $x_t = e_x x_t^{(0:\bar{k})}$. Therefore, the learning process is different because it includes both the direct effect and endogenous response of the interest rate to the expectations hierarchy through the the responses of the inflation and output.

The following proposition shows how each firm i's expectation about the hierarchy of expectations is formed, in addition to the average expectation.

Proposition 2. *Given the guessed expectation hierarchy (19) and the observational equation (26), firm i's rational expectation about the expectations hierarchy is given by*

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = \left(I_k - \bar{K}C\right) A E_{i,t-1}\left[x_{t-1}^{(0:\bar{k})}\right] + \bar{K}CAx_{t-1}^{(0:\bar{k})} + \bar{K}CB\varepsilon_t + \bar{K}Dv_{it}. \tag{28}$$

The average first-order expectation of $x_t^{(0:\bar{k})}$ is such that

$$E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = \left(I_k - \bar{K}C\right) A E_{t-1}^{(1)}\left[x_{t-1}^{(0:\bar{k})}\right] + \bar{K}CAx_{t-1}^{(0:\bar{k})} + \bar{K}CB\varepsilon_t, \tag{29}$$

where $\bar{K}$ is the steady-state Kalman gain, which can be computed by solving the Riccati equation that combines the following equations:

$$\begin{aligned} \bar{K} &= \bar{P}C'\left[C\bar{P}C' + D\Sigma_v D'\right]^{-1}, \\ \bar{P} &= A\left(I_k - \bar{K}C\right)\bar{P}A' + B\Sigma_\varepsilon B' \end{aligned} \tag{30}$$

The proof is presented in Appendix A.2. It also shows in detail how, given the dynamics of first-order expectations from equation (29), one can also verify the guessed hierarchy of expectations dynamics from equation (19) by finding the equilibrium A and B matrices.

The steady-state Kalman gain matrix $\bar{K}$ provides the optimal stationary response of the expectations hierarchy to changes in the observables i_t and s_{it}. This matrix depends on not only the ratio σ_η/σ_π but also the noise-to-signal ratio σ_v/σ_π.

As in any DSGE model, the equilibrium response of the endogenous variables depends on the persistence of the shocks. In both the ICI, the responses also depend on the persistence of the expectations about the shocks whereas, in the ICK model, they also depend on the persistence of higher-order average expectations.

The dispersed information model exhibits an interesting interaction between the response of endogenous variables to the expectations hierarchy and expectations formation. Specifically, the endogenous variables depend on expectations about their future, which, in turn, are a function of the future expectation hierarchy. Thus, the response Q depends on the persistence of the higher-order expectations, A. Furthermore, since firms observe the interest rate, the dynamics of the hierarchy of expectations depends on the reaction of the endogenous variables to the hierarchy itself (through the matrix C), i.e., the matrix A depends on Q. Therefore, there is a feedback loop between responses to higher-order expectations and their formation, which is not the case in the imperfect information model since the expectation formation is exogenous in that model.

4 Inflation-output trade-off and inflation target expectations

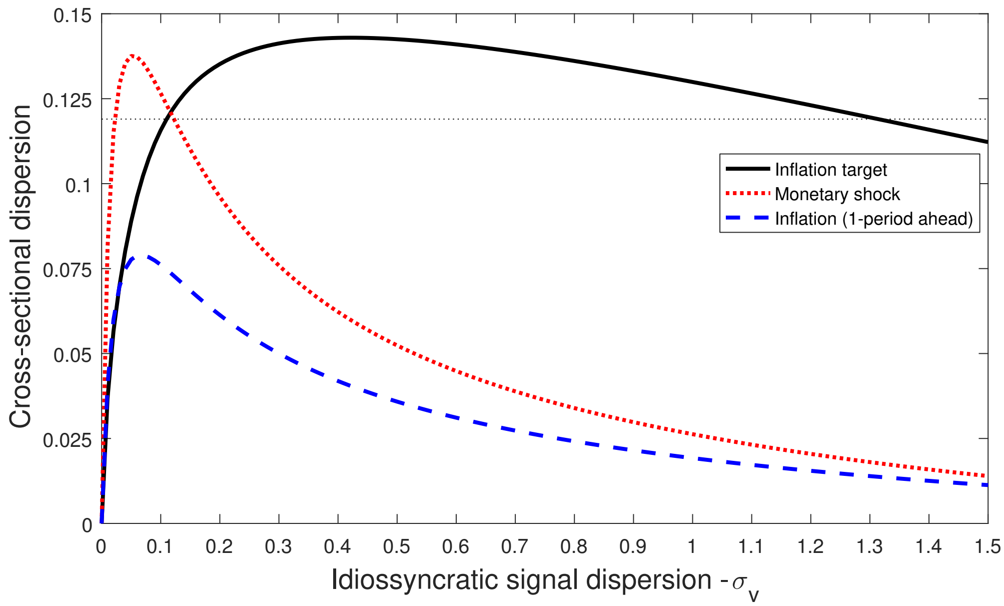
This section presents how the learning about the inflation target affects the trade-off between inflation and output for two policies: a disinflation policy and a contractionary monetary shock. I compare the impulse response functions (IRFs) of inflation and output of the full information (FI), imperfect common information (ICI) and imperfect common knowledge (ICK) models in response to the monetary or inflation target shocks.

The parameters are calibrated using standard values. The discount factor, $\beta = 0.99$, is such that annual interest rate is approximately 4%. The probability of not optimizing prices, $\theta = 2/3$, is set such that firms on average keep prices fixed by 3 quarters. Log utility is assumed ($\gamma = 1$), and the inverse Frisch elasticity is set to be consistent with the macro literature ($\varphi = 2$). Moreover, the elasticity of substitution is chosen such that the firm's mark-up in the steady-state is 11% ($\varepsilon = 10$), following Woodford (2003). The labor share, α, is calibrated as 0.7, consistent with national account data of developed countries. The calibration of those parameters implies weak complementarity in pricing decisions ($\xi = 0.81$). For the monetary policy rule, I use moderate responses to inflation and output ($\phi_\pi = 1.5$, $\phi_y = 0.25$).

The persistence parameters for the inflation target process and monetary shocks are calibrated as $\rho_\pi = 0.975$ and $\rho_m = 0.5$, respectively. Therefore, deviations from the steady-state target are calibrated to be quite persistent, implying a long-lasting change in monetary policy. In contrast, monetary shocks represent short-lived variation in the interest rate. For instance, the half-life for the former is 27 quarters, whereas that of the latter is 1 quarter. Moreover, for the standard deviations of shocks, I use common estimated values ($\sigma_\eta = 0.15$ and $\sigma_\pi = 0.05$) for medium-scale models, both under perfect information (Adolfson et al.; 2007) and imperfect information (Del Negro and Eusepi; 2011). This calibration implies a ratio $\sigma_\eta/\sigma_\pi = 3$.

The dispersion parameter σ_v is not a standard one and requires more effort to calibrate than the others. From equation (28), it is easy to see that v_{it} is the source of heterogeneity of expectations about the hierarchy of expectations. Then, the dispersion in expectations is driven by the variance of the idiosyncratic signal. The unconditional (steady-state) cross-sectional covariance of the expectations hierarchy can then be computed by solving the Lyapunov equation

Figure 2: Cross-sectional standard deviation as function of σ_v



Note: The dotted horizontal line refers to the target value of 0.119.

$$\Sigma = (I_k - \bar{K}C)A\Sigma A'(I_k - \bar{K}C)' + \bar{K}_s\bar{K}_s'\sigma_v^2, \tag{31}$$

where $\bar{K}_s$ is the steady-state Kalman gain related to the private signal. Formally, $\Sigma \equiv var\left(E_{it}[x_t^{(0:\bar{k})}]\right) = E\left[\left(E_{it}[x_t^{(0:\bar{k})}] - E_t^{(1)}[x_t^{(0:\bar{k})}]\right)\left(E_{it}[x_t^{(0:\bar{k})}] - E_t^{(1)}[x_t^{(0:\bar{k})}]\right)'\right]$. By the definition of x_t, the second element of the diagonal is the cross-sectional variance of the inflation target expectations. Moreover, given the solution of the model from equation (18), one can see that the cross-sectional covariance for the endogenous variables can be computed as $Q\Sigma Q'$.

Owing to lack of data regarding expectations about the target, I use as a proxy the longest (10-year ahead) forecast about inflation available from the Survey of Professional Forecasters (SPF). For the period 1991Q1-2015Q4, the quarterly cross-sectional standard deviation is 0.119. It turns out, however, that the relationship between Σ and σ_v is non-monotonic. Note that as σ_v increases, the Kalman gain $\bar{K}_s$ decreases since agents optimally choose to react less to noisier signals. Therefore, the second term in equation (31) responds non-monotonically to increases in the private signal dispersion. Figure 2 plots the standard deviation of the (first-order) expectation of the inflation target, monetary shock and 1-period ahead inflation as functions of σ_v.[15] The dotted horizontal line refers to the target value of 0.119 from the SPF data.

One can see that two values of σ_v (approximately 0.1 and 1.3) are consistent with the target

[15]I show the range values $\sigma_v \in [0, 1.5]$ for expositional convenience. For higher values, as σ_v increases, the cross-sectional dispersion slowly decreases towards zero.

value. The baseline calibration, $\sigma_v = 0.1$, is chosen for two reasons. First, as Figure 2 shows, values of σ_v near 0.1 yield the highest dispersion in expectations about both 1-quarter ahead inflation and monetary shocks. In the SPF data, the cross-sectional standard deviation for the 1-quarter ahead inflation forecast, for the same period, is approximately 0.20. The model can generate a standard deviation equal to approximately 40% of the one in the data, but it would be unwise to think that only one source of dispersion should be able to fully replicate the disagreement of inflation expectations in the data. Second, the baseline calibration implies a ratio $\sigma_v/\sigma_\pi = 2$, whereas the other option leads to a sufficiently high value that firms almost do not react to the private signal to form expectations about the target.

4.1 Effects of monetary shocks

This subsection discusses how the differences in expectations formation described in the previous section matter for the effects of monetary shocks. Figure 3 shows the IRFs of endogenous variables and the monetary shock and inflation target expectations after a monetary shock for the three aforementioned models. In the bottom panels, the solid blue line shows the actual shock (full information expectation), the dotted red lines show the ICI expectation for both shocks, and the dashed marked black lines show the hierarchy of expectations (up to third order) of the ICK model.[16]

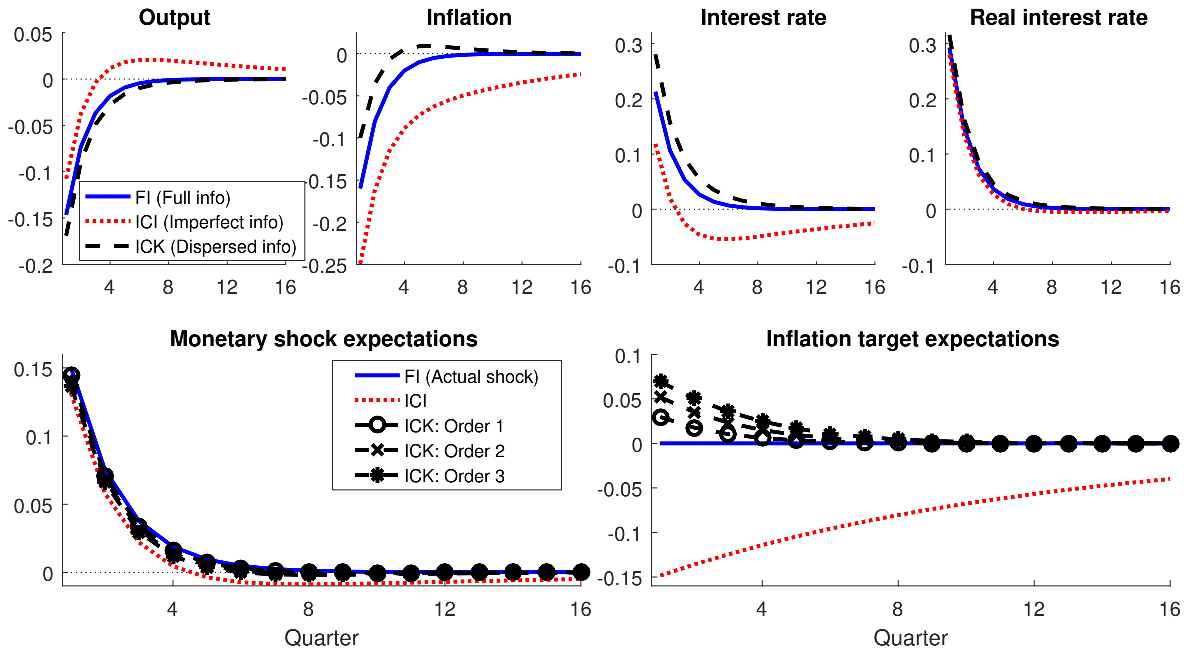
For all models, a positive monetary shock induces lower inflation expectations that decrease inflation and increase the ex-ante real interest rate trajectory, thereby weakening output. More interestingly, the ICI model has a better trade-off between inflation and output than the full information model – as inflation decreases more with fewer costs in terms of output – whereas the opposite happens for the ICK model.

This difference between models in terms of the inflation-output trade-off depend crucially on the differences in the learning process about the inflation target shown in the bottom-right panel. Specifically, for the ICI model, firms beliefs are that inflation target decreased, whereas for the ICK model, the average expectation is that it increased (and average higher-orders are even higher). This implies that inflation expectations – relative to the perfect information benchmark – will be lower for the ICI model and higher for ICK model, thus inducing a relatively higher inflation in the latter. Therefore, in the latter, the central bank sets relatively higher interest rate, leading to higher output costs.

The contrast in the learning process about the inflation target is a reflection of the differences in the signal extraction problem explained in the previous section. That is, the ICK model has to consider the endogenous effects of the hierarchy of expectations about monetary shocks and

[16]For the model solution, I use $\bar{k} = 10$, which is more than sufficient for an accurate solution. The graph shows only the first three orders for presentation purposes.

Figure 3: Impulse response function after a monetary shock



Note: Response to a one standard-deviation shock. Inflation, interest rates and inflation target expectations are annualized and expressed in percentage points.

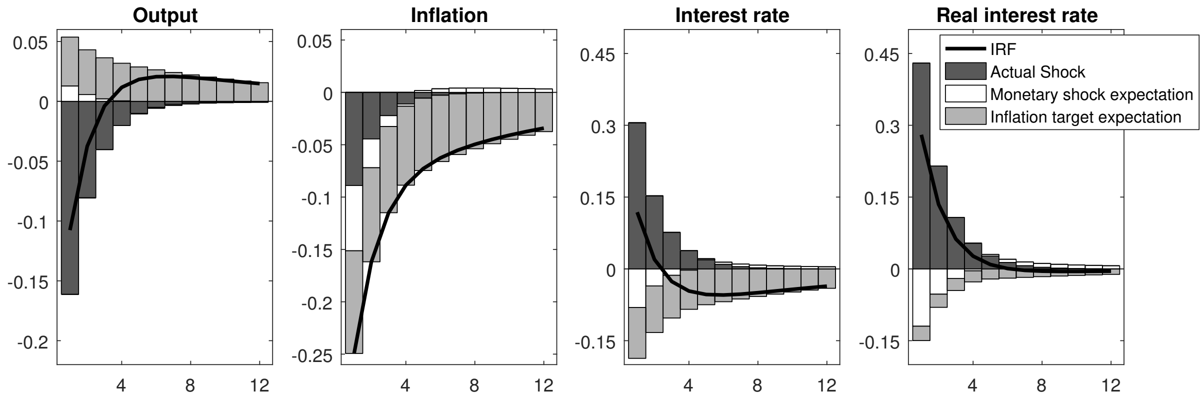
the inflation target on the interest rate. In other words, firms have to learn about the economy to infer the monetary policy stance. Specifically, firms have to consider whether the interest rate has increased as a response to higher inflation and output, which in turn might be a consequence of a negative monetary shock, or a higher target. For monetary shocks, the endogenous effect is dominant (dominated) when forming expectations about the inflation target (monetary policy).

The intuition is as follows. Because the inflation target is assumed to be a very persistent process, its effect on inflation and output is strong and long-lasting. Then, firms' optimal filtering imply that the increase in the interest rate is more likely to be an outcome of an increase in inflation and output caused by a higher inflation target than a direct effect from a lower inflation target. For monetary shocks, which are much less persistent, the endogenous effect is dominated by the direct effect.[17] Thus, firms expect that the monetary shock was positive. Therefore, the assumption that ρ_π is close to one is key for this result. In section 4.3, I present a sensitivity analysis of the results performed by perturbing key parameters.

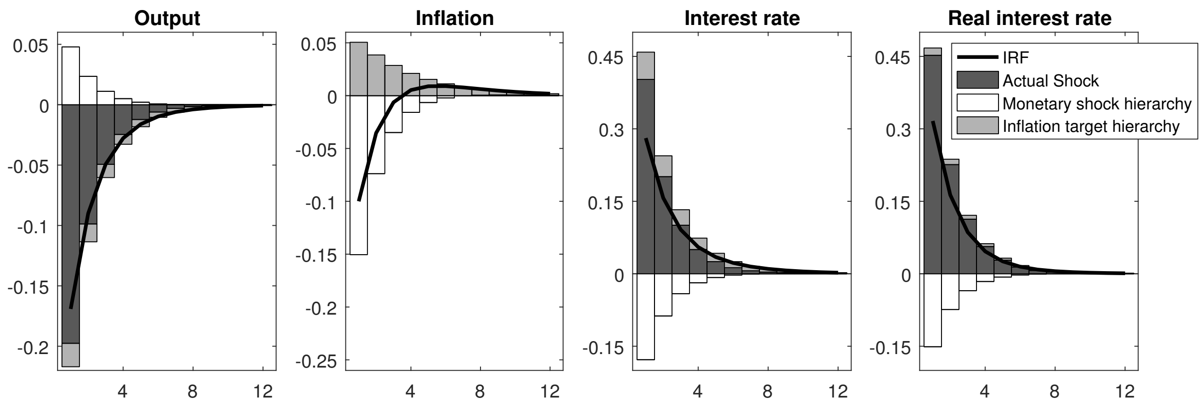
Figure 4 emphasizes the impacts of the expectation formation differences on each endogenous variable. Specifically, the IRFs (black solid lines) are decomposed into the contribution of the monetary shock, its expectations, and the inflation target expectations (and all the hierarchy of

[17] Galí (2008, chapter 3) shows analytically the relation between the persistence of the monetary shock and the dominance of the direct effect for the benchmark New Keynesian model, i.e., the nominal interest rate increases after the shock.

Figure 4: IRFs after a monetary shock: expectations hierarchy decomposition of IRFs



(a) ICI model



(b) ICK model

Note: Response to a one standard-deviation monetary shock. Inflation, interest rates and inflation target expectations are annualized and expressed in percentage points. Appendix A.4 shows how to compute this decomposition.

expectations for the ICK model). Panel (a) shows the results for the ICI model, and panel (b) shows those for the ICK model. Appendix A.4 discusses how to compute this decomposition based on the model solution and the expectations dynamics.

In the ICI model, as shown by the dark gray bar, a monetary shock increases the interest rate and decreases inflation expectations, leading to lower inflation and a higher real interest rate (since $\phi_\pi > 1$). This implies a decrease in output, which reinforces the decrease in inflation. Moreover, firms expect a positive monetary shock and a lower inflation target (recall from Figure 3). The white and light gray bars show that expectations about both shocks intensify the decrease in inflation. This has a negative impact on both the nominal and real interest rate through the endogenous part of the Taylor rule, thereby leading to a positive impact on output according to the Euler equation. The reinforcing effect on inflation is sufficiently strong to push the interest rate below the steady-state value in the third quarter after the shock.

Panel (b) shows two important differences between the ICK and ICI models. First, whereas the effects of the actual shock and its (higher-order) expectations have similar effects as their counterparts in the ICI model, one qualification is important. Since output is not observable by the firms in the ICK model, only expectations about output matter, instead of output itself (compare equations (14) and (15)). Therefore, the actual shock has no impact on inflation, and the decrease in inflation is caused only by expectations about a positive monetary shock. Second and more interestingly, the inflation target expectations have the opposite effect than in the ICI model. Since firms, on average, expect a higher inflation target, this expectation counterbalances the negative effect of a higher interest rate on inflation through relatively higher inflation expectations. This leads the central bank to increase even further the interest rate owing to the endogenous effect of inflation, which reinforces the decrease in output.

In summary, firms in the ICK model expect that a higher inflation target, leading to a worse inflation-output trade-off than in the full information model since inflation expectations are relatively higher. The opposite is true for the ICI model: the trade-off is better than in the full information model because inflation expectations are lower.

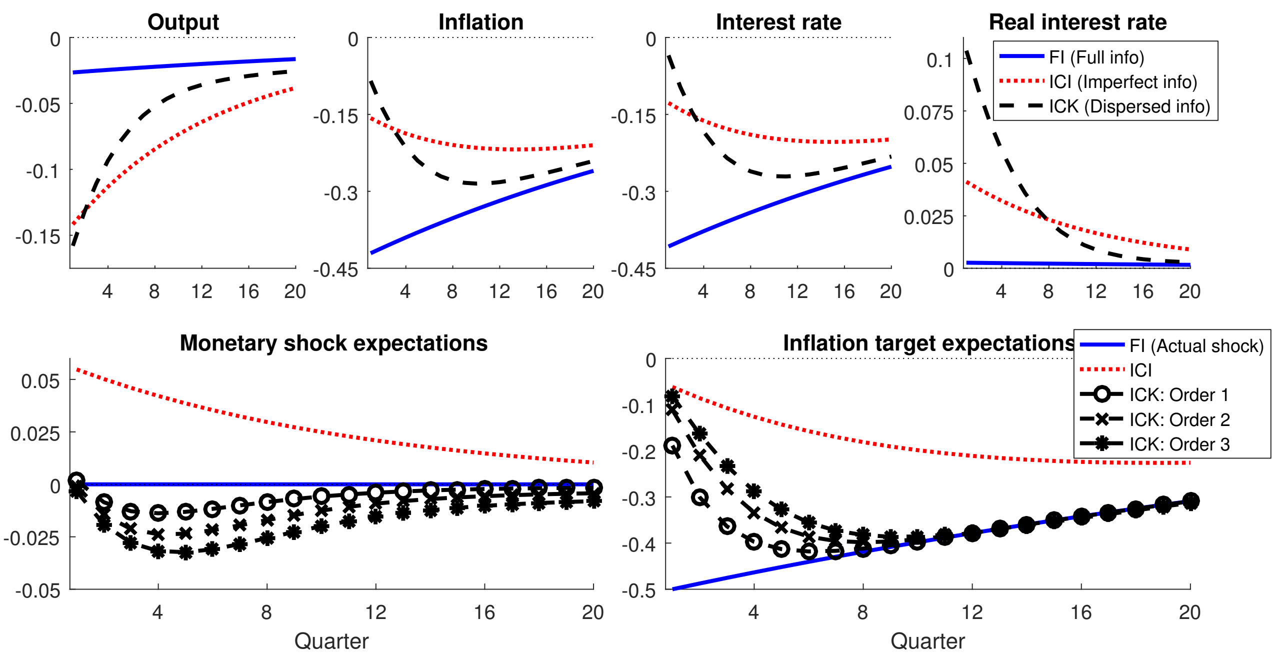
4.2 Disinflation period: a lower inflation target

Now I turn to the relation of disinflation costs and the learning process of the target. Figure 5 shows the IRFs for a decrease in the inflation target. Note that the inflation target is highly persistent, but the process is still stationary; thus, all variables will eventually return to their steady-state values. More importantly, as emphasized by Mankiw and Reis (2002), Erceg and Levin (2003) and others, the full information model is unable to present realistic costs in terms of output for disinflations.[18] In this setting, the decrease in inflation target is known by firms, who adjust their inflation expectations accordingly, which implies that real interest rate exhibits a negligible increase that leads to essentially no output costs.

Both the ICI and ICK models can account for the costs in terms of output of the disinflation because firms have to learn that the inflation target has decreased. Therefore, inflation expectations do not decrease as much as in the full information model, implying relatively higher real interest rates leading to a decrease in output. For the baseline calibration, it is clear that there is a worse trade-off between inflation and output in the ICI model than in the ICK model, as inflation is relatively higher and output relatively lower for almost all the time after the change in the target. This is the case because the learning process about the target is much faster in the ICK model, as shown in the bottom-right panel of Figure 5. This depends crucially on the fact that σ_v/σ_π is relatively small, consistent with the data regarding long-term inflation expectations. Intuitively,

[18] Ball (1994b) provide evidence of costs of disinflation for a panel of countries.

Figure 5: Impulse response function after a decrease in the inflation target



Note: Response to a 0.5 p.p. decrease in the inflation target. Inflation, interest rates and inflation target expectations are annualized and expressed in percentage points.

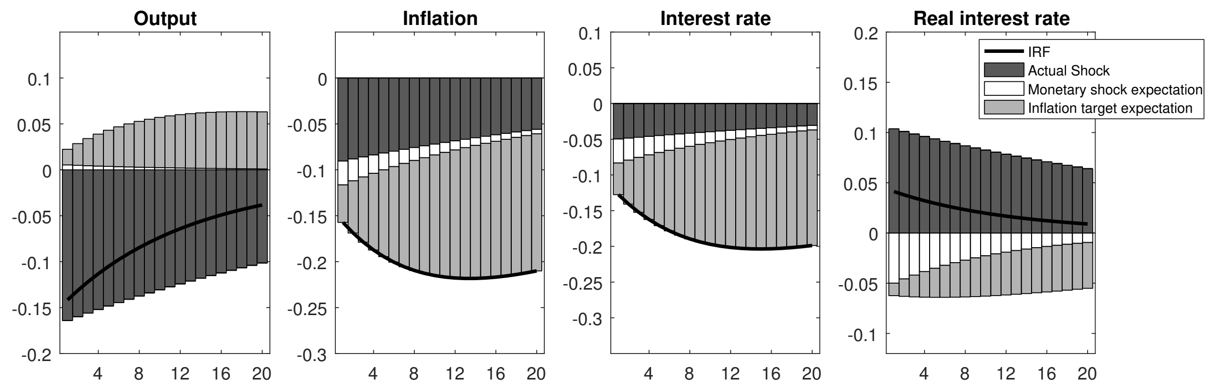
the costs of disinflation arise from the fact that real interest rates are relatively higher since inflation expectations slowly incorporate the fact that the inflation target decreased. Then, smaller σ_v/σ_π corresponds to better-informed firms and lower output costs.

Moreover, for the target shock, the learning about the inflation target are similar whereas the qualitative differences are in the learning process are for the monetary shock (see bottom panels of Figure 5). This difference is the opposite of the one from the monetary shock shown above.

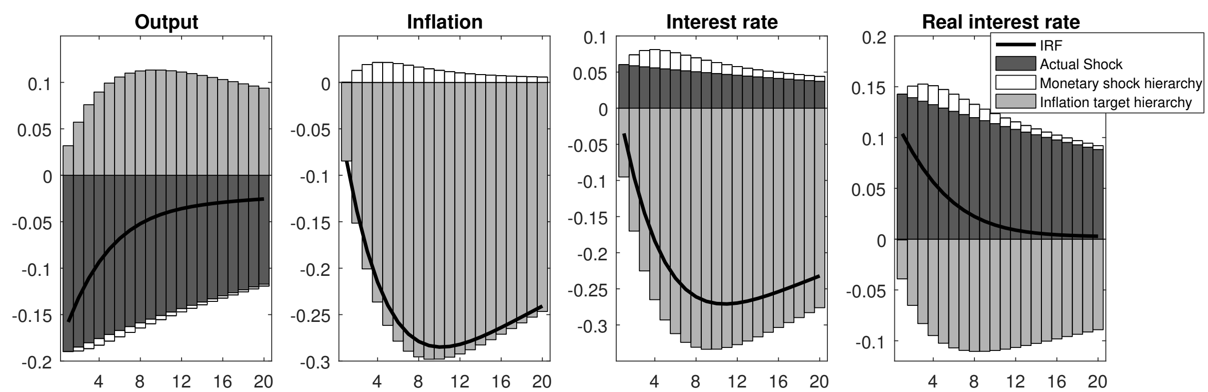
The reason is the following. For the monetary shock, both interest rate and the adjusted interest rate increase after the shock. For the (negative) target shock, the endogenous effect is strong enough to drive the interest rate below steady-state for both models (see Figure 5). Despite the decrease in the interest rate, the adjusted interest rate, $\hat{i}_t$, increases (see equation (21)). Then, ICI and ICK model extract information from variables that go in different directions – the opposite situation from the monetary shock –, which implies a switch in which shocks the qualitative differences in learning happens.

In other words, for the ICI model, firms expect that the increase in the adjusted interest rate is a result of a lower inflation target or a positive monetary shock. For the ICK model, the direct effect is dominant for monetary shocks, and the endogenous effect dominates for the inflation target. For the latter, firms find it more likely that the interest decrease is an endogenous response of inflation and output to a lower inflation target than a direct effect of a higher target. The opposite happens for the expectations of monetary shocks, except for the initial impact, since the initial decrease in the interest rate is relatively small. Firms find it more likely that the decrease in interest rate is a direct effect of a negative monetary shock than an endogenous decrease in output and inflation owing to a positive monetary shock.

Figure 6: IRFs after a decrease in inflation target: expectations hierarchy decomposition



(a) ICI model



(b) ICK model

Note: Response to a decrease of 0.5 p.p. in the inflation target. Inflation, interest rates and inflation target expectations are annualized and expressed in percentage points. Appendix A.4 shows how to compute this decomposition.

In contrast with monetary shocks, the difference in learning does not play an important role for the target shock. The expectation hierarchy decomposition of the IRFs from Figure 6 illustrate the reason for this. In panel (a) it is clear that the expectation about a positive monetary shock (white bar) induces both lower inflation and real interest rate that leads to a small positive effect on output. In the panel (b), firms on average expect a negative monetary shock that has a positive effect on inflation and the real interest rate (since $\phi_\pi > 1$). Therefore, whereas in the former, expectations about the monetary shock reinforce the effect of the shock, they do the opposite in the latter. However, those effects are rather small to counterbalance the effects of the decrease in the inflation target. This is the case because the inflation target shock is very persistent, whereas monetary shocks are short-lived.

The models deliver a counterfactual empirical prediction that in disinflations, the nominal interest rate should decrease (although both correctly capture the increase in real rates). The ICK model performs relatively better than the ICI model in this sense because the interest rate almost does not change on impact and gradually decreases after a target shock. In imperfect information models with richer dynamics, such as that of Erceg and Levin (2003), this problem is corrected, which might be also true for the ICK model.

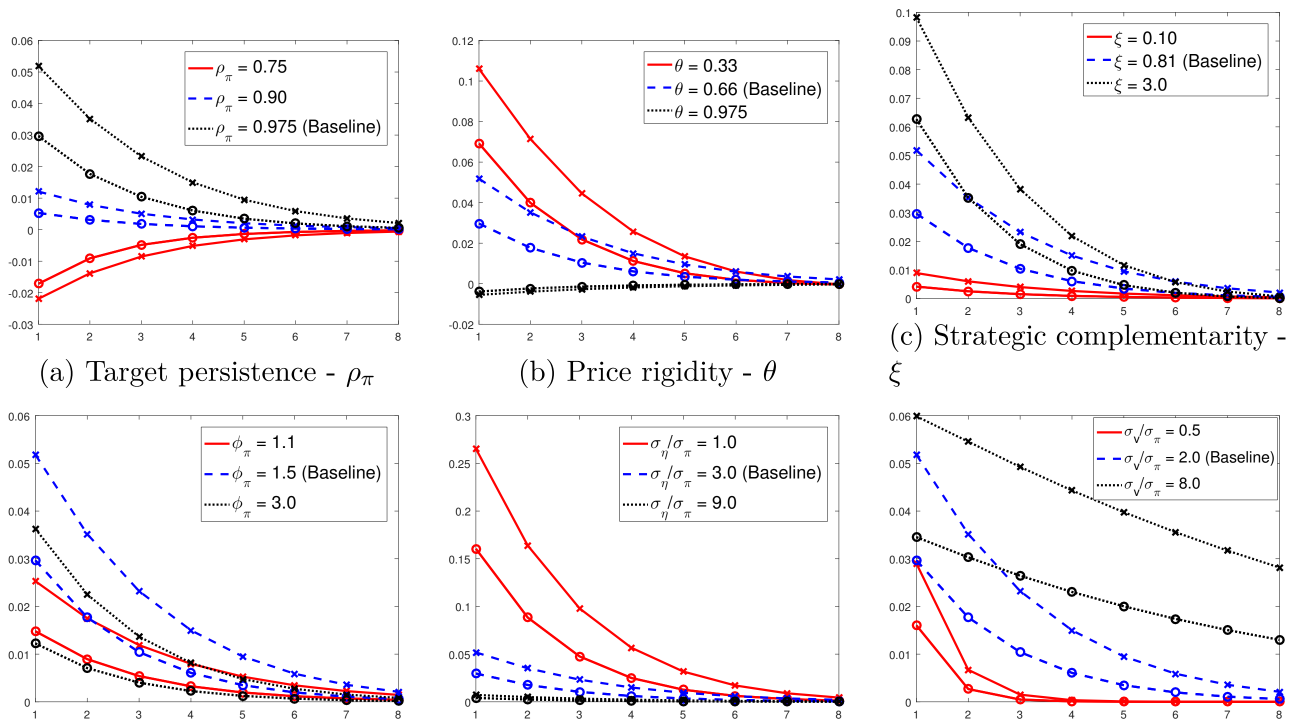
4.3 Strategic uncertainty and learning from endogenous variables

The introduction of the noisy signal, s_{it}, affects the signal extraction in two aspects. It provides additional information about the inflation target but also prevents firms from observing the current inflation and output. Previously, I argued that the key difference between the models is the fact that firms need to consider the endogenous effect of shocks when forming expectations. In this section, I provide two different evaluations. First, a sensitivity analysis of the results is performed to check whether the difference in the learning process about the target remains when key parameters are perturbed. Second, by imposing a counterfactual information set to firms, I demonstrate that the main difference in the learning is not a result of the additional information that the signal s_{it} itself provides but rather the fact that firms cannot infer the output and inflation from their own decisions.

Figure 7 shows the IRFs of the first-order (dashed lines with "$\circ$" marker) and second-order (dashed lines with "$\times$" marker) expectations about the inflation target after a monetary shock. Each panel shows the counterpart of the bottom-right plot in Figure 3 for different parameters. All plots show the IRFs under three different calibrations for each parameter, keeping all other parameters fixed to their baseline values.

Plot (a) shows that the expectations about the inflation target are positive as long as the persistence about the target is sufficiently high. Higher persistence of the target implies a higher effect of expectations about the target on inflation and output, i.e., a more pronounced endogenous effect on the Taylor rule. This increases the likelihood that the rise in the interest rate was due to an higher target. Plot (b) shows that for most of the calibrations, firms expect that the inflation target increases after a monetary shock. Only when prices are extremely rigid ($\theta = 0.975$, which implies that prices are fixed, on average, for 40 quarters) this is not the case. Intuitively, the more rigid prices are, the flatter the Phillips curve becomes, implying weaker effects of the shocks to the interest rate on inflation. Therefore, firms find it less likely that the interest rate increased as a result of higher inflation. For all other parameters shown in Figure 7, the expectations are consistent with the baseline result, even for extreme calibrated values. Moreover, they are all consistent with the endogenous part of the interest rate rule leading to expectations of an increase in the target. Panel (c) shows that both strong strategic complementarities ($\xi = 0.1$) or substitutability ($\xi = 3$) in pricing decisions are consistent with expectations of an increase in the target. Stronger strategic complementarity in pricing decisions is associated with higher expectations about the target since the Phillips curve becomes flatter.

Figure 7: Sensitivity analysis: IRFs of inflation target expectations after a monetary shock



(a) Target persistence - ρ_π (b) Price rigidity - θ (c) Strategic complementarity - ξ

(d) Inflation response - ϕ_π (e) Std. deviation ratio - σ_η/σ_π (f) Std. deviation ratio - σ_v/σ_π

Note: Markers "∘" and "×" denote average first-order and second-order expectations, respectively. Responses to a one standard-deviation monetary shock. Inflation target expectations are annualized and expressed in percentage points.

Panel (d) shows that the parameter that controls the response of the interest rate to inflation has an interesting non-monotonic effect on target expectations. For relatively low values of ϕ_π, a higher response leads to higher inflation target expectations (compare the red lines with the blue ones). Instead, for relatively high values, a higher response leads to lower target expectations (compare the blue lines with the black ones). Intuitively, a higher ϕ_π increases the endogenous effect because it amplifies the response of the interest rate to inflation, but it also decreases the effects of both shocks on inflation because firms' expectations take into account that the central bank is more aggressive towards inflation. The second effect dominates as the value of ϕ_π become high.

The last two parameters shown are the noise-to-signal ratios. An higher ratio σ_η/σ_π (panel (e)) implies a smaller increase in expectation about the target. In words, when the monetary shock is relatively noisier than the target shock, the interest rate provides less reliable information. Thus, firms rely relatively more on their private signal, which, on average, indicates that the target is unchanged. Panel (f) shows that the higher noise-to-signal ratio, σ_v/σ_π, the more pronounced is the increase in the inflation target expectations . This is the case since, on average, the information from the private signal is that the target is unchanged, whereas the information from the interest rate is that the target is increased. Therefore, the noisier the information from the private signal, the more strongly firms will weight the information from the interest rate and thus on average expect a higher target.

In summary, the results shown in Figure 7 reinforce that the key reason for the differences in the expectations formation is the high persistence of the target. Even extreme deviations of other parameters from the baseline calibration still sustain the qualitative learning differences about the inflation target after a monetary shock.[19] It is worth noting that only the persistence parameters (ρ_π and ρ_η) and the noise-to-signal ratio (σ_η/σ_π) affect the learning process of the ICI model, as in any standard signal extraction problem. In the ICK model, however, all parameters affect the learning process because they influence the response of the interest rate to shocks and their hierarchy of expectations through the endogenous part of the rule.

Now I turn to the relative importance of the additional information about the target from the private signal vis-a-vis the lack of information about inflation and output induced by the strategic uncertainty to explain the results. To do so, consider the following thought experiment. Impose that firms in the ICK model observe the adjusted interest rate $\hat{i}_t$ instead of i_t. Therefore, the information set of each firm i given by

$$\hat{\mathcal{I}}_t^i = \{\hat{i}_\tau, s_{i\tau}, \pi_{\tau-1}, y_{\tau-1} | \tau \geq t\}, \tag{32}$$

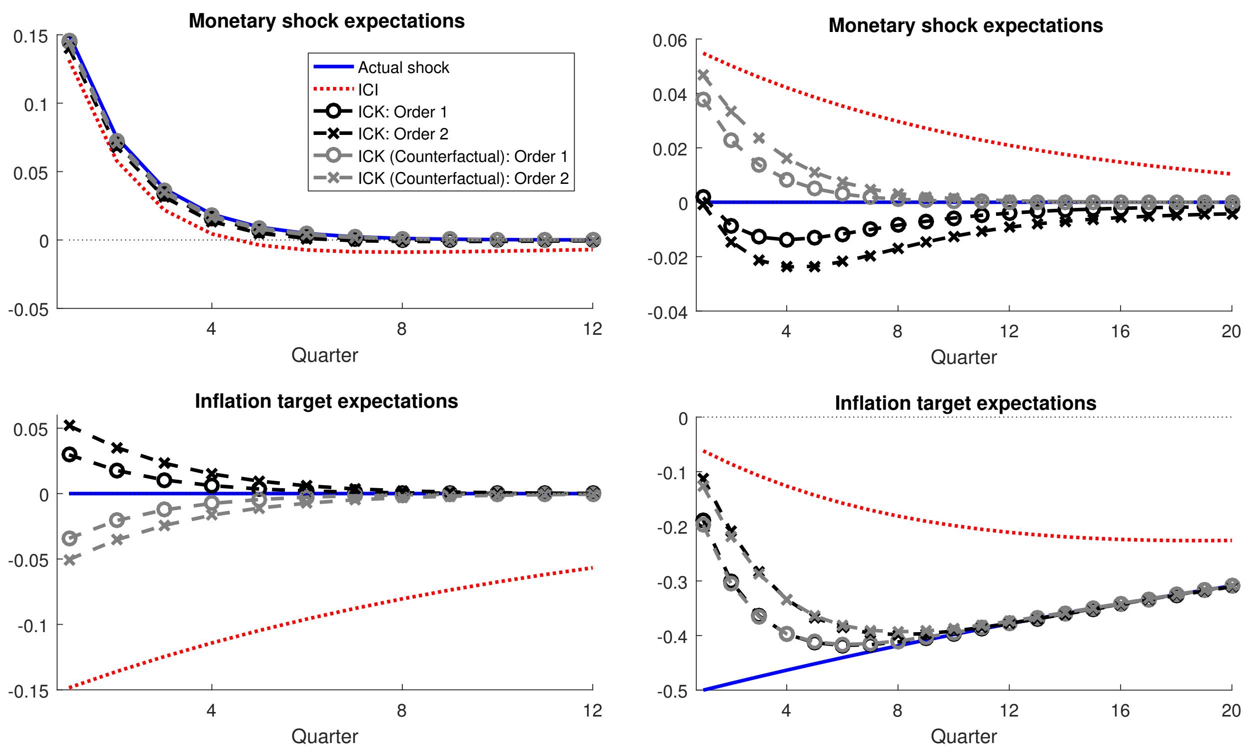
instead of the one in equation (24). I refer to $\hat{\mathcal{I}}_t^i$ as the counterfactual information set in the following. Note that this assumption is not the same of imposing that firms observe inflation and output since, in that case, it would affect the Phillips curve (14).[20] In other words, firms have additional information from the private signal without losing any information from the interest rate. In such case, the state equation is the expectations hierarchy from equation (19), and the observation equations are equations (21) and (10).

Figure 8 compares the IRFs of the expectations about the shocks under the counterfactual information set with the true information sets after a monetary shock (panel (a)) and after a

[19] Evidently, one can pick a combination of parameters with an extreme calibration that attenuates the endogenous forces of the monetary rule leading to a similar learning process between the ICK and ICI models.

[20] Imposing this stronger version leads to similar results.

Figure 8: Counterfactual information set: IRFs of monetary shock and inflation target expectations



(a) Monetary shock (b) Inflation target shock

Note: Panel (a) shows the responses to a one standard-deviation monetary shock. Panel (b) shows the responses to a 0.5 p.p. decrease in the inflation target. Inflation, interest rates and inflation target expectations are annualized and expressed in percentage points.

decrease in the inflation target (panel (b)). In both panels, it is clear that the qualitative learning difference vanishes when one considers the counterfactual information set for the ICK model. Under this modified information set, firms expectations respond in a similar manner as in the ICI model but are more accurate. Moreover, the additional information from the private signal does not help to explain the qualitative differences in learning about the shocks – rather, the opposite is true. For the monetary shock, the learning differences are in terms of the inflation target. The private signal, on average, indicates that the target did not change and thus implies *lower* inflation target expectations for the ICK (recall the discussion of panel (f) of Figure 7), unlike the benchmark case. Moreover, for the target shock, the public signal indicates a target decrease, implying that firms find it less likely that the interest rate decreased because of a negative monetary shock. This goes in the opposite direction of explaining the expectations about the target in the baseline ICK model.

Therefore, the key mechanism through which expectations differ is the change in capability to extract information from the interest rate induced by the strategic uncertainty, not the additional information about the target from the private signal.

5 Transparency and the effectiveness of monetary policy

This section discusses how transparency regarding the inflation target can affect the effectiveness of the monetary policy and demonstrates the importance of those learning differences in this particular context. Suppose that firms also observe a noisy public signal about the inflation target given by

$$s_t^p = \pi_t^* + u_t, \tag{33}$$

where $u_t \sim \mathcal{N}(0, \sigma_u^2)$. One interpretation of this public signal is that it represents the communication from central bank about its goals. In that case, σ_u is a measure of the degree of (lack of) transparency. Following Faust and Svensson (2002), if $\sigma_u = 0$, firms observe the true inflation target. In that case, the central bank is fully transparent and both ICI and ICK models are equivalent to the full information model. For a sufficiently high σ_u ($\sigma_u \to \infty$), the equilibrium with the public signal is exactly the same as one without the signal because it provides no useful information about the target. In that case, the central bank is fully opaque. Finally, for any intermediate value of σ_u, the model is subject to imperfect transparency.

This modeling device is a simple and reduced form to define transparency in terms of the accuracy of the communication of the central bank about its actual target π_t^* (recall that the steady-state target Π^* is common knowledge). An alternative interpretation is that σ_u controls the confidence bands around the inflation target, which is related to the common practice of central banks under inflation-targeting regimes. Of course, in the model, the public signal is centered around the time-varying inflation target, whereas in practice, the bands are defined around the announced fixed inflation target. On the other hand, we cannot know for sure whether the central bank is aiming at any point within the band or the announced target itself.

To calibrate this parameter, I choose $\sigma_u/\sigma_\pi = 2.5$. This implies that the public information is less informative than the private information. This calibration (equivalent to $\sigma_u = 0.125$) corresponds to an approximately 1.0 annual percentage (95% confidence) band. However, the results are qualitative and hold for any value of σ_u that provides meaningful additional information for the firms.

The additional information is straightforward to implement in the ICI model because one simply has to include the public signal in the observational equation (21), and the solution method is the same. For the ICK model, however, the solution method requires two important modifications. First, the higher-order expectations will also depend on the noise of the public signal. Second, when forming individual expectations, firms have to take into account that the public signal affects the average expectation.

Formally, the guessed dynamics for the higher-order expectations is such that

$$x_t^{(0:\bar{k})} = Ax_{t-1}^{(0:\bar{k})} + B_\varepsilon \varepsilon_t + B_u u_t. \tag{34}$$

Moreover, by adding the public signal to the observational equation (26), the observational equation can be written as

$$\begin{bmatrix} i_t \\ s_{it} \\ s_t^p \end{bmatrix} = \begin{bmatrix} Q_i \\ e_{\pi^*} \\ e_{\pi^*} \end{bmatrix} x_t^{(0:\bar{k})} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} v_{it} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_t = Cx_t^{(0:\bar{k})} + D_v v_{it} + D_u u_t, \tag{35}$$

where Q_i and e_{π^*} are defined as before and C, D_v and D_u are defined accordingly.

The following proposition shows how expectations about the hierarchy of expectations are formed. It extends the Proposition 2 by including public signals as observables.

Proposition 3. *Given the guessed expectation hierarchy (34) and the observational equation (35), firm i's rational expectation about the expectations hierarchy is given by*

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = \left(I_k - \bar{K}C\right) AE_{i,t-1}\left[x_{t-1}^{(0:\bar{k})}\right] + \bar{K}CAx_{t-1}^{(0:\bar{k})} + \bar{K}CB_\varepsilon\varepsilon_t + \bar{K}\left(CB_u + D_u\right)u_t + \bar{K}D_v v_{it}. \tag{36}$$

The average first-order expectation of $x_t^{(0:\bar{k})}$ is such that

$$E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = \left(I_k - \bar{K}C\right) AE_{t-1}^{(1)}\left[x_{t-1}^{(0:\bar{k})}\right] + \bar{K}CAx_{t-1}^{(0:\bar{k})} + \bar{K}CB_\varepsilon\varepsilon_t + \bar{K}\left(CB_u + D_u\right)u_t \tag{37}$$

where $\bar{K}$ is the steady-state Kalman gain, which can be computed by solving the Riccati equation that combines the following equations:

$$\begin{aligned} \bar{P} &= A\left[\bar{P} - \bar{K}\left(\bar{P}C' + \Sigma_{cov}\right)'\right]A' + \Sigma_{state}, \\ \bar{K} &= \left[\bar{P}C' + \Sigma_{cov}\right]\left[C\bar{P}C' + \Sigma_{obs}\right]^{-1}, \end{aligned} \tag{38}$$

where $\Sigma_{obs} = D_v\Sigma_v D_v' + D_u\Sigma_u D_u'$, $\Sigma_{state} = B_\varepsilon\Sigma_\varepsilon B_\varepsilon' + B_u\Sigma_u B_u'$ and $\Sigma_{cov} = B_u\Sigma_u D_u'$.

The proof is in the Appendix A.2. It also shows in detail how, given the dynamics of first-order expectations from equation (37), one can also verify the guessed hierarchy of expectations dynamics from equation (19) by finding the equilibrium A, B_ε and B_u matrices.

Note that whereas the idiosyncratic noise v_{it} is washed-out in the average expectations, the noise of the public signal, u_t, remains. Therefore, u_t is an additional source of variation for endogenous variables but only through changes in average expectations. For instance, Lorenzoni (2009), and Angeletos and La'O (2010) discuss the importance of those noise shocks about productivity to explain business cycles fluctuations.[21]

When using the Kalman filter, firms have to take into account the correlation of errors of the state and observational equations (u_t appears in both equations). This is why the term Σ_{cov} appears in the equation (38). Intuitively, when forming expectations about higher-order average expectations, agents take into account that public signals also affect other agents' expectations. This result relates to Morris and Shin (2002), which show that agents with strategic complementarities tend to overreact to public signals vis-a-vis private signals relative to the reaction expected by their relative precision.

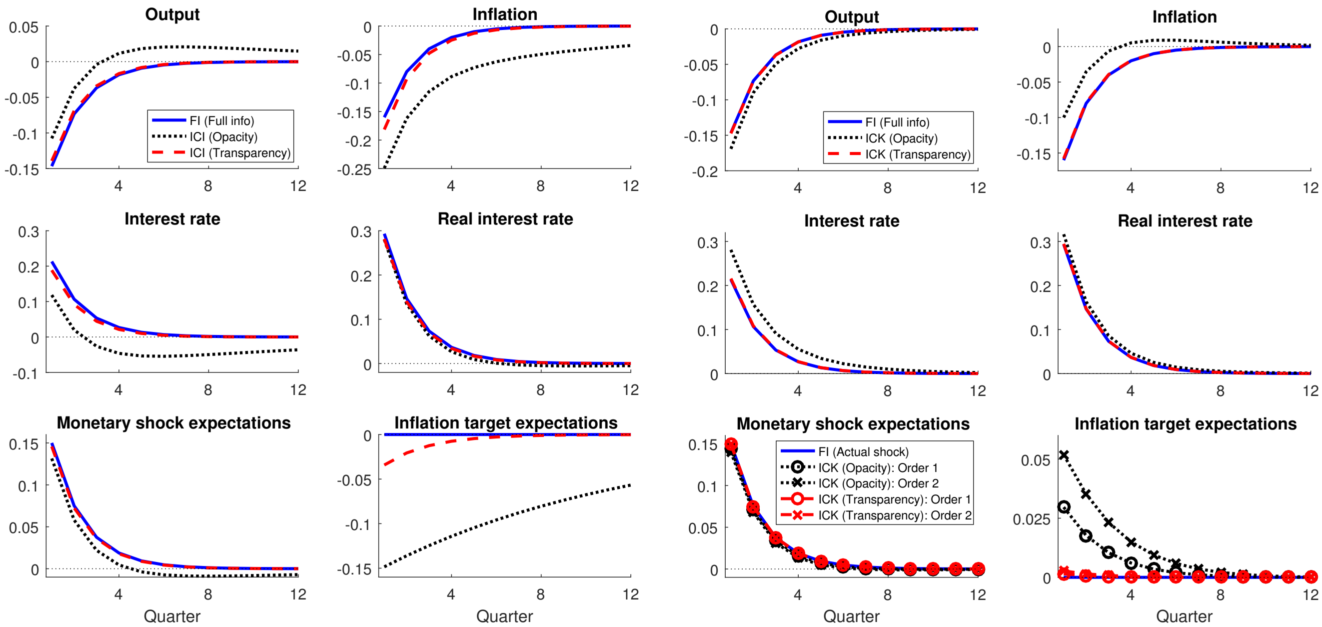
Figure 9 compares the IRFs after a monetary shock under imperfect transparency (with a public signal) and under opacity (no public signal). Panels (a) and (b) show the IRFs for the ICI and ICK models, respectively. For both models, one can see that the additional information from the public signal is sufficiently precise to induce more accuracy of firms' expectations and faster learning towards the true values of the shocks. How fast firms learn about the inflation target depends on σ_η/σ_π and σ_u/σ_π for the ICI model and also on σ_v/σ_π for the ICK model.

In summary, both models become closer to the full information model as the public information about the target becomes more accurate. For the ICI model, this implies *higher* inflation expectations than without the public signal, since firms expect a lower inflation target than otherwise. Then, the central bank responds with stricter tightening in the interest rate, leading to a sharper decrease in output with higher inflation. In that case, higher transparency would lead to a *worse* inflation-output trade-off. In contrast, in the ICK model, firms expect on average that the inflation target increased. The information from the public signal implies that firms lower their expectations about the target. This results in *lower* inflation expectations than otherwise and thus lower inflation with lower output costs. Therefore, higher transparency implies a *better* inflation-output trade-off. Thus, differences in the expectations formation play an important role in assessments of the effects of transparency on the effectiveness of the monetary policy.

Figure 10 shows the same plot for the case of a decrease of 0.5% in the inflation target. For both the ICI and ICK models, higher transparency diminishes uncertainty about the inflation target such that expectations about the target are lower and converge faster to the true value of the target than otherwise. For the baseline calibration, the effect of transparency is more pronounced in the ICI model since it takes longer for firms to learn about the target in this model. For both models, because of the forwarding-looking nature of the Euler equation, output costs are lower under transparency – despite the initially higher real interest rate – since the accumulated real

[21] One difference from those papers is that I do not impose the simplifying assumption that period $t-1$ shocks become common knowledge at period t.

Figure 9: Transparency and inflation-output trade-off: IRFs after a monetary shock



(a) ICI model (b) ICK model

Note: Response to a one standard-deviation shock. Inflation, interest rates and inflation target expectations are annualized and expressed in percentage points.

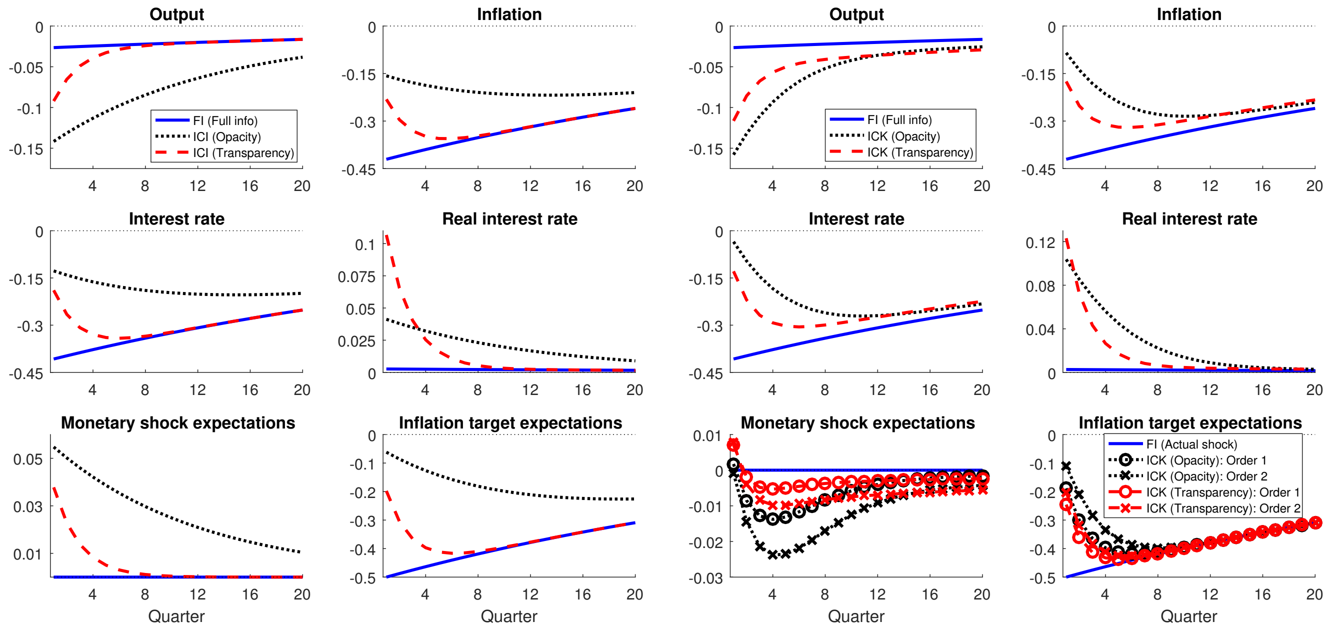
interest rate during the whole period is lower. Therefore, a more transparent monetary policy in terms of the inflation target results in less costly disinflation.

5.1 Discussion

By adding a public noisy signal to a standard imperfect information model that the recent literature used for disinflation periods, I study the effects of transparency on the effectiveness of monetary policy. Whereas for disinflation periods, more transparency implies lower costs in terms of output, this is not the case for monetary shocks. This difference is particularly troublesome since the focus on transparency exhibited by central banks is not restricted to central banks that are pursuing disinflation policies. Rather, this trend toward more transparency is a widespread feature of modern central banking.

Surprisingly, the effect of higher transparency on the trade-off between inflation and output depends on the details of the learning process, even with a small change in the information available to firms – i.e., the introduction of a private noisy signal about the target. For the disinflation case, this modification affects the results only quantitatively, but it changes the results qualitatively for the monetary shock case. Specifically, it changes the conclusions about the relationship between the transparency about the target and the effectiveness of monetary policy. For the dispersed

Figure 10: Transparency and inflation-output trade-offs under disinflation



(a) ICI model (b) ICK model

Note: Response to a 0.5 p.p. decrease in the inflation target. Inflation, interest rates and inflation target expectations are annualized and expressed in percentage points.

information model, irrespective of which type of shock is considered, more transparency results in a lower inflation-output trade-off.

A priori, one would think that the introduction of a noisy idiosyncratic signal should not be key to answer the question on which I focus here. However, the private signal not only provides an additional information about the target but also changes the nature of the learning process from the interest rate. This change leads to a richer and more realistic learning process about the inflation target and the monetary policy shock. The uncertainty about monetary policy that firms face translates into uncertainty about the economy, which, in turn, introduces a confounding factor into the interest rate. This affects formation of expectations about monetary policy. This type of consideration seems consistent with the fact that central banks communicate to explain the reasons for their interest rate decisions, which are usually related to developments in inflation and economic activity. I believe that this feedback interaction about learning about the economy and learning the reasons for a change in the interest rate is a better representation of the challenges that forecasters face than the one from the standard signal extraction problem based on the imperfect information model. In other words, the standard imperfect information model imposes that agents have too much information about the state of the economy. This matters when they are extracting information from endogenous variables since the fact that they know that other agents' decisions are exactly like theirs might provide some useful information about those variables.

6 Concluding remarks

In this paper, I study whether a central bank's transparency about its goals increases the effectiveness of the monetary policy in terms of the inflation-output trade-off. If the widespread view of central banks that transparency is an important tool to control inflation with lower output costs is correct, then the imperfect common knowledge model provides a better explanation of the relationship between transparency and the effectiveness of monetary policy. In order study transparency within the imperfect common knowledge framework, this paper also builds on Nimark (2008) and Melosi (2017) to extend the solution method of the imperfect common knowledge model to include exogenous public signals.

Strategic uncertainty induced by the noisy idiosyncratic signal generates interesting and richer learning about the monetary policy stance. Agents must form expectations about economic developments to understand the reasons for the monetary authority's decisions, whereas the imperfect information model has a mechanical learning process from an exogenous signal extraction problem. I argue that the former provides a better description of the real uncertainty that agents face when a central bank changes the interest rate.

This paper abstracts from the uncertainty about real shocks that might interact with the expectations formation about the monetary shocks, as discussed by Melosi (2017). Furthermore, the reasons for the inflation target change, and the optimal monetary and communication policies are not explored. Those are interesting and challenging avenues for future research.

A Proofs

A.1 Proof of Proposition 1

Using the Kalman filter with the state equation as (22) and observational equation (21), one can write the update equation as

$$E_t(x_t) = E_{t-1}(x_t) + K_t[\hat{\imath}_t - E_{t-1}(\hat{\imath}_t)], \tag{A.1}$$

where the Kalman gain, K_t, and mean square error of the one-step ahead prediction error, $P_{t+1|t}$, have the following dynamics:

$$\begin{aligned} K_t &= P_{t|t-1}C_1' \left[C_1 P_{t|t-1} C_1'\right]^{-1} \\ P_{t+1|t} &= A_1(I_n - K_t C_1) P_{t|t-1} A_1' + \Sigma_\varepsilon. \end{aligned} \tag{A.2}$$

Since x_t is stationary and Σ_ε is positive definite, then there exists a steady-state solution such that $\tilde{P} = P_{t+1|t} = P_{t|t-1}$ which implies the steady-state Kalman gain $\tilde{K} = \tilde{P}C_1' \left[C_1 \tilde{P} C_1'\right]^{-1}$ (see Hamilton; 1995, chap. 13). Therefore, taking expectations of $\hat{\imath}_t$ and using the fact that $E_{t-1}(x_t) = A_1 E_{t-1}(x_{t-1})$, the dynamics of $E_t(x_t)$ can be written as equation as

$$E_t(x_t) = (I_n - \tilde{K}C_1)E_{t-1}(x_{t-1}) + \tilde{K}\hat{\imath}_t. \tag{A.3}$$

Moreover, substituting (21) into the equation above and using equation (22) one can find the equation (23) in the Proposition 1.

In the more general setup, when the public signal is included in the information set of the firms, the observation equation becomes

$$\begin{bmatrix} \hat{\imath}_t \\ s_t^p \end{bmatrix} = \begin{bmatrix} 1 & -(\phi_\pi - 1) \\ 0 & 1 \end{bmatrix} x_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_t = C_1 x_t + D u_t, \tag{A.4}$$

where C_1 and D are defined accordingly. Following the same steps

$$E_t(x_t) = (I_n - \tilde{K}C_1)A_1 E_{t-1}(x_{t-1}) + \tilde{K}C_1 A_1 x_{t-1} + \tilde{K}C_1\varepsilon_t + \tilde{K}Du_t, \tag{A.5}$$

where $\tilde{K} = \tilde{P}C_1' \left[C_1 \tilde{P} C_1'\right]^{-1} + D\Sigma_u D'$ and $\tilde{P} = A_1(I_n - \tilde{K}C_1)\tilde{P}A_1' + \Sigma_\varepsilon$ are the steady-state Kalman gain matrix and the mean square error of the prediction error of the state, respectively. □

A.2 Proof of Propositions 2 and 3

In this subsection, I show the proof of the Proposition 3 and discuss that the results of Proposition 2 is a special case when there is no public signals in agents' information set.

Consider firm i with the observational equation (35), restated by convenience:

$$Z_{it} = Cx_t^{(0:\bar{k})} + D_v v_{it} + D_u u_t, \tag{A.6}$$

where $Z_{it} = [\ i_t \quad s_{it} \quad s_t^p\ ]'$ and C, D_v and D_u are defined as before. In general, if agents observe endogenous variables, is a C linear transformation of Q, such as $C = c_0 + c_1 Q$, where c_0 and c_1 are conformable matrices. The proof here works for any observational equation such that Z_{it} is a $q \times 1$ vector, C is a $q \times k$ matrix, D_v is a $q \times p_v$ matrix and D_u is a $q \times p_u$ matrix. Note that if $D_u = 0_{q \times p_u}$, this equation is in the same form as the baseline observation equation (26). Moreover, firms are forming expectations about the hierarchy of expectations that is given by the state equation (34), also restated:

$$x_t^{(0:\bar{k})} = Ax_{t-1}^{(0:\bar{k})} + B_\varepsilon \varepsilon_t + B_u u_t. \tag{A.7}$$

In the following, I show that if $D_u = 0_{q \times p_u}$, then $B_u = 0_{k \times p_u}$, i.e., the state equation becomes the same as in the baseline case (equation (34)). Therefore, this proof works for both Propositions 2 and 3.

Proof. Each firm i uses the Kalman filter and find the update equation given by

$$E_{i,t}\left[x_t^{(0:\bar{k})}\right] = E_{i,t-1}\left[x_t^{(0:\bar{k})}\right] + K_t\left[Z_{it} - E_{i,t-1}\left[Z_{it}\right]\right], \tag{A.8}$$

where K_t is the Kalman gain given by

$$K_t = \left[P_{t/t-1}C' + \Sigma_{cov}\right]\left[CP_{t/t-1}C' + \Sigma_{obs}\right]^{-1}, \tag{A.9}$$

where $\Sigma_{obs} = D_v \Sigma_v D_v' + D_u \Sigma_u D_u'$ and and $\Sigma_{cov} = B_u \Sigma_u D_u'$. As usual, the mean squared error (MSE) of the one-period ahead prediction error is given by

$$P_{t+1/t} = A\left[P_{t/t-1} - K_t\left[CP_{t/t-1} + \Sigma_{cov}\right]\right]A' + \Sigma_{state}, \tag{A.10}$$

where $\Sigma_{state} = B_\varepsilon \Sigma_\varepsilon B_\varepsilon' + B_u \Sigma_u B_u'$. For details of this deviation, see for instance Hamilton (1995, chap. 13).

Using the observational equation, (A.6), taking expectations and inserting in (A.8) one can find:

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = E_{i,t-1}\left[x_t^{(0:\bar{k})}\right] + K_t\left[Cx_t^{(0:\bar{k})} + D_v v_{it} + D_u u_t - CE_{i,t-1}\left[x_t^{(0:\bar{k})}\right]\right] \tag{A.11}$$

Therefore, one can rewrite the equation above as

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = \left(I_k - K_t C\right) E_{it-1}\left[x_t^{(0:\bar{k})}\right] + K_t\left[Cx_t^{(0:\bar{k})} + D_v v_{it} + D_u u_t\right] \tag{A.12}$$

where $k = n(\bar{k}+1)$. Using the fact that $E_{i,t-1}\left[x_t^{(0:\bar{k})}\right] = AE_{i,t-1}\left[x_{t-1}^{(0:\bar{k})}\right]$ and substituting equation (A.7), one can find:

$$E_{it}\left[x_t^{(0:\bar{k})}\right] = \left(I_k - K_t C\right) AE_{i,t-1}\left[x_{t-1}^{(0:\bar{k})}\right] + K_t CAx_{t-1}^{(0:\bar{k})} + K_t C(B_\varepsilon \varepsilon_t + B_u u_t) + K_t[D_v v_{it} + D_u u_t] \tag{A.13}$$

I follow the literature by focusing in the stationary equilibrium. Therefore, the expectation of each individual i in the stationary equilibrium is the one which the MSE is in steady-state, i.e., firms update their forecast based on the steady-state Kalman gain. In other words, the dynamics of expectations depends only in the properties of the process they are forecasting and do not depend in the period t. Using equations for $P_{t+1/t}$, $P_{t/t}$ and K_t one can find the Riccatti equation

$$P_{t+1/t} = A\left[P_{t/t-1} - P_{t/t-1}C'\left[CP_{t/t-1}C' + \Sigma_{obs}\right]^{-1}\left[CP_{t/t-1} + \Sigma_{cov}\right]\right]A' + \Sigma_{state} \tag{A.14}$$

Therefore, one need to iterate this equation to find the steady-state MSE, $\bar{P}$, and compute its counterpart Kalman gain, $\bar{K}$. Nimark (2017) shows that if is x_t stationary process, then the expectations hierarchy about this process, $x_t^{(0:\bar{k})}$, is also stationary. This and the fact that Σ_ε is positive definite, then there exists a steady-state solution such that $\bar{P} = P_{t+1|t} = P_{t|t-1}$ which implies the steady-state Kalman gain $\bar{K} = K_t = K_{t-1}$ (see Hamilton; 1995, chap. 13).

The individual expectation in equation of Proposition 3 is one in equation (A.13) above using the steady-state Kalman gain, $\bar{K}$. Moreover, the first order expectation is easily computed by:

$$\begin{aligned} E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] &\equiv \int_0^1 E_{it}\left[x_t^{(0:\bar{k})}\right] di \\ &= \left(I_k - \bar{K}C\right) AE_{t-1}^{(1)}\left[x_{t-1}^{(0:\bar{k})}\right] + \bar{K}CAx_{t-1}^{(0:\bar{k})} + \bar{K}CB_\varepsilon\varepsilon_t + \bar{K}(CB_u + D_u)u_t \end{aligned} \tag{A.15}$$

□

Now I turn to verify the claim that the expectations hierarchy has dynamics guessed in equation

(34). It is straightforward to see that $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = x_t^{(1:\bar{k}+1)}$. By definition, any order higher than $\bar{k}$ is not relevant for the equilibrium. Then, without loss of generality, I can set $E_t^{(s)}[x_t] = 0$ if $s > \bar{k}$. Therefore, one can rewrite $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]$ as

$$E_t^{(1)}\left[x_t^{(0:\bar{k})}\right] = \begin{bmatrix} x_t^{(1:\bar{k})} \\ x_t^{(\bar{k}+1)} \end{bmatrix} = \begin{bmatrix} x_t^{(1:\bar{k})} \\ 0_{n\times 1} \end{bmatrix} = \begin{bmatrix} 0_{n\bar{k}\times n} & I_{n\bar{k}} \\ 0_{n\times nk} & 0_{n\times n\bar{k}} \end{bmatrix} \begin{bmatrix} x_t^{(0:k-1)} \\ x_t^{(k:\bar{k})} \end{bmatrix} = T x_t^{(0:\bar{k})}, \tag{A.16}$$

where in the second equality I used the definition of $x_t^{(1:\bar{k}+1)}$ and in the third equality I used that $E_t^{(\bar{k}+1)}[x_t] = 0$, without loss of generality. In the last equality, T is defined accordingly. Therefore, the first-order expectation of expectations hierarchy is a linear transformation of the expectations hierarchy itself, given by the matrix T. By the same token, $x_t^{(0:\bar{k})}$ can be rewritten as a linear combination of x_t and $E_t^{(1)}\left[x_t^{(0:\bar{k})}\right]$ such that

$$x_t^{(0:\bar{k})} \equiv \begin{bmatrix} x_t \\ x_t^{(\bar{1}:\bar{k})} \end{bmatrix} = \begin{bmatrix} I_n \\ 0_{n\bar{k}\times n} \end{bmatrix} x_t + \begin{bmatrix} 0_{n\times n\bar{k}} & 0_{n\times n} \\ I_{n\bar{k}} & 0_{n\bar{k}\times n} \end{bmatrix} \begin{bmatrix} x_t^{(1:\bar{k})} \\ x_t^{(\bar{k}:\bar{k}+1)} \end{bmatrix} = e_x' x_t + T' E_t^{(1)}\left[x_t^{(0:\bar{k})}\right], \tag{A.17}$$

where e_x is defined in the same way as before. Substituting the first order expectation from equation (A.15) into the identity above:

$$x_t^{(0:\bar{k})} \equiv e_x x_t + T'\left(I_k - \bar{K}C\right) A E_{t-1}^{(1)}\left[x_{t-1}^{(0:\bar{k})}\right] + T'\bar{K}CAx_{t-1}^{(0:\bar{k})} + T'\bar{K}CB_\varepsilon \varepsilon_t + \bar{K}(CB_u + D_u)u_t \tag{A.18}$$

Then, substituting equation (22) and using fact that $x_t = e_x x_t^{(0:\bar{k})}$ one can find that

$$x_t^{(0:\bar{k})} \equiv e_x'\left(A_1 e_x x_{t-1}^{(0:\bar{k})} + \varepsilon_t\right) + T'\left(I_k - \bar{K}C\right) A E_{t-1}^{(1)}\left[x_{t-1}^{(0:\bar{k})}\right] + T'\bar{K}CAx_{t-1}^{(0:\bar{k})} + T'\bar{K}CB_\varepsilon\varepsilon_t + T'\bar{K}(CB_u + D_u)u_t, \tag{A.19}$$

Using the order transformation from equation (A.16) at period $t-1$ and rearranging I find that

$$x_t^{(0:\bar{k})} = \left[e_x' A_1 e_x + T'\left(I_k - \bar{K}C\right) AT + T'\bar{K}CA\right] x_{t-1}^{(0:\bar{k})} + \left[T'\bar{K}CB_\varepsilon + e_x'\right]\varepsilon_t + \left[T'\bar{K}(CB_u + D_u)\right] u_t. \tag{A.20}$$

Therefore, the expression above verify that $x_t^{(0:\bar{k})}$ follows the guessed form and the square brackets terms provide identities for A, B_ε and B_u such that

$$
\begin{aligned}
A &= e_x A_1 e_x' + T' \left(I_k - \bar{K} C \right) AT + T' \bar{K} C A \\
B_\varepsilon &= T' \bar{K} C B_\varepsilon + e_x' \\
B_u &= T' \bar{K} (C B_u + D_u)
\end{aligned}
\tag{A.21}
$$

Finally, by the last equation if $D_u = 0_{q \times p_u}$, the fixed point solution for B_u is $0_{k \times p_u}$. This means that in the case of no public noise signals, the expectation hierarchy process becomes the same as in equation (19). Moreover, if $D_u = 0_{q \times p_u}$ is a matrix of zeros, $\Sigma_{cov} = 0_{k \times q}$. Therefore, the steady-state Kalman gain from Proposition Proposition 3 becomes the same as in the Proposition 2.

A.3 Model solution

The imperfect common knowledge model consists in a the Euler equation, the Taylor rule and the Phillips curve (equations 11, 4 and 14, respectively). They can be easily written as the following system of linear equations

$$
F_1 \mathbb{E}_t \left[Y_{t+1} \right] + F_2 E_t^{(1)} \left[Y_{t+1} \right] + G_1 Y_t + G_2 E_t^{(1)} \left[Y_t \right] + L_1 E_t^{(1)} \left[x_{t+1} \right] + L_2 E_t^{(1)} \left[x_t \right] + M_1 x_t = 0, \tag{A.22}
$$

where Y_t is a $m \times 1$ vector of endogenous variables and x_t is a $n \times 1$ vector of *unobservable* exogenous shocks. $\mathbb{E}_t[\cdot]$ is the full information expectation operator and $E_t^{(1)}[\cdot]$ is the average expectation of agents with private information.

Here for convenience I rewrite the guessed solution

$$
Y_t = Q x_t^{(0:\bar{k})}. \tag{A.23}
$$

Using the standard undermined coefficients method, one can find a solution for Q. First, computing the full information expectation of Y_{t+1} such that

$$
\begin{aligned}
\mathbb{E}_t \left[Y_{t+1} \right] &= Q \mathbb{E}_t \left[x_{t+1}^{(0:\bar{k})} \right] \\
&= Q A x_t^{(0:\bar{k})},
\end{aligned}
\tag{A.24}
$$

where in the last equality it was used the fact that $\mathbb{E}_t \left[E_t^{(k)} \left[x_t \right] \right] = E_t^{(k)} \left[x_t \right]$ for any k.[22] By computing the individual expectation and integrating on i one can find the average expectation from agents with idiosyncratic information such as

[22]This come from the fact that the information set of the full information agents contain the information set of private information agents. One can think in the full information agent as a one that receives all private signals, which would reveal the unobservable shocks x_t.

$$E_t^{(1)}[Y_t] = QE_t^{(1)}\left[x_t^{(0:\bar{k})}\right]$$
$$= QTx_t^{(0:\bar{k})}, \tag{A.25}$$

where the last equality uses the order transformation matrix that relates the expectation of hierarchy to expectations hierarchy itself. Then, one can find the average expectation for endogenous variables in $t+1$:

$$E_t^{(1)}[Y_{t+1}] = QE_t^{(1)}\left[x_{t+1}^{(0:\bar{k})}\right]$$
$$= QATx_t^{(0:\bar{k})}, \tag{A.26}$$

where the last equality uses that $E_t^{(1)}\left[x_{t+1}^{(0:\bar{k})}\right] = AE_t^{(1)}\left[x_t^{(0:\bar{k})}\right]$. Substituting the expectations (A.24-A.26) in the equation (A.22) one can find:

$$\left[G_1Q + F_1QA + G_2QT + F_2QAT + (L_1A_1 + L_2)e_x'T + M_1e_x'\right)\right] x_t^{(0:\bar{k})} = 0_{m\times n} \tag{A.27}$$

Since this equality is valid for any value of $x_t^{(0:\bar{k})}$, the only way it is always satisfied is if the term in square brackets is equal to zero. The solution for Q can be found by straightforward vectorization. Note, however, that unlike the full information model and the standard imperfect information model, the solution Q depends on A which in turn is a function of Q through the matrix C of the observation equation. Since firms observe an endogenous variable, the observational equation depends on the response of endogenous variables to the expectations hierarchy (recall equations 26 and 35). Therefore, the model solution is a fixed point of the solution to equations (A.27) and (A.21).

Following a similar approach of Nimark (2008) and Melosi (2017), the fixed-point solution can be found by the algorithm:

Set the initial values $(A^{(0)}, B_\varepsilon^{(0)}, B_u^{(0)})$ and a small tolerance $\epsilon > 0$ and set $i = 1$. Then, follow the steps:

1. given $A = A^{(i-1)}$, solve for Q the equation (A.27) by vectorization. Set $Q^{(i)} = Q$.
2. Given $Q^{(i)}$, construct the matrix C from the observation equation (A.6). Set $C^{(i)} = C$.
3. Given $B_\varepsilon^{(i-1)}$, $B_u^{(i-1)}$ and $C^{(i)}$ compute the Kalman gain, K_t and $P_{t+1|t}$ using equations (A.9) and (A.10), respectively. Set $K^{(i)} = K_t$ and $P^{(i)} = P_{t+1|t}$.
4. Given $B_\varepsilon^{(i-1)}$, $B_u^{(i-1)}$, $A^{(i-1)}$, $C^{(i)}$ and $K^{(i)}$, compute the B_ε, B_u, A using equation (A.21). Set $B_\varepsilon^{(i)} = B_\varepsilon$, $B_u^{(i)} = B_u$, $A^{(i)} = A$.

5. if min $\|B_{\varepsilon}^{(i)} - B_{\varepsilon}^{(i-1)}\| < \epsilon$, $\|B_{u}^{(i)} - B_{u}^{(i-1)}\| < \epsilon$, $\|A^{(i)} - A^{(i-1)}\| < \epsilon$ and $\|P^{(i)} - P^{(i-1)}\| < \epsilon$, stop iterating or else set $i = i + 1$ and go back to step 1.

A.4 Decomposition of Impulse Response Functions

For convenience, I rewrite the solution of the ICI model given by

$$Y_t = Q_1 x_t + Q_2 E_t[x_t]. \tag{A.28}$$

by the definition of x_t, one can rearrange this equation as

$$Y_t = Q_{1,\eta}\eta_t + Q_{1,\pi^*}\pi_t^* + Q_{2,\eta}E_t[\eta_t] + Q_{2,\pi^*}E_t[\pi_t^*], \tag{A.29}$$

where Qi,j are conformable submatrices of Q_i such that $i = \{1,2\}$ and $j = \{\pi^*, \eta\}$. Therefore, this four terms perfectly decompose the response of endogenous variables to shocks. For instance, in the decomposition of the impulse response function a standard deviation monetary shock shown in Figure 4, I set $\varepsilon_0^{\eta} = \sigma_{\eta} = 0.15$, $\varepsilon_t^{\eta} = 0$ for $t \geq 1$ and $\varepsilon_t^{\pi^*} = 0$ for all t. Then, shocks and their expectations evolve accordingly to equations (22) and (23), respectively. Then, first term standards for the "Actual shock", the second equals zero, the third for "Monetary shock expectations" and the last one for "Inflation shock expectations". For the inflation target shock it works analogous.

For the ICK model, for convenience, I write the model solution given by

$$Y_t = Q x_t^{(0:\bar{k})} \tag{A.30}$$

by the definition of x_t and $x_t^{(0:\bar{k})}$ one can rearrange this equation as

$$Y_t = Q_{0,\eta}\eta_t + Q_{0,\pi^*}\pi_t^* + \sum_{k=1}^{\bar{k}} Q_{k,\eta}E_t^{(k)}[\eta_t] + \sum_{k=1}^{\bar{k}} Q_{k,\pi^*}E_t^{(k)}[\pi_t^*] \tag{A.31}$$

where Qk,j are conformable submatrices of Q such that $k = 1, 2, ..., \bar{k}$ and $j = \{\pi^*, \eta\}$. Again, this four terms decompose the response to shocks. Then, decomposition of Figure 4, the innovations are defined as describe above and shocks and their higher-order expectations evolve accordingly to equations (22) and (19), respectively. Then, first term standards for the "Actual shock", the second equals zero, the third for "Monetary shock hierarchy" and the last one for "Inflation shock hierarchy". For the inflation target shock it works analogous. I chose to decompose the IRFs in only three terms: the impact of the actual shock, of all higher-order expectations to the monetary shock and all higher-order expectations to the target shock. One could alternatively decompose it also by each order of each shock.

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