Information in the Yield Curve: A Macro-Finance Approach*

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Summary

We use a macro-finance model incorporating macroeconomic and financial factors to study the term premium in the U.S. bond market. Estimating the model using Bayesian techniques, we find that one factor is responsible for most of the variation in bond premia. Furthermore, the model-implied bond premia explain up to 40% of the variability of one- and two-year excess returns. Using the model to decompose yield spreads into an expectations and a term premium component, we find that, although this decomposition does not seem important to forecast economic activity, it is crucial to forecast inflation for most forecasting horizons.

**JEL classifications:** E43; E44; E47

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1 Introduction

The term structure of interest rates has long been recognized as a potential source of information about future macroeconomic developments. This prevalent belief on the forward-looking characteristic of the yield curve is best represented by the expectations hypothesis (EH). According to this theory, the slope of the yield curve reflects market expectations of the average future path of short-term interest rates. Following the EH, it makes sense then to use yield curve information to forecast macroeconomic aggregates such as real economic activity and inflation.\footnote{Estrella (2005) investigates the theoretical reasons behind the predictive power of the yield curve to forecast output and inflation. Regarding the prediction of economic activity, see, among others, Estrella and Hardouvelis (1991), Estrella and Mishkin (1998), Plosser and Rouwenhorst (1994), and Stock and Watson (1989). For inflation, see, for example, Pindyck (1990), Mishkin (1990), Estrella and Mishkin (1997), and Lorrion and Mishkin (1991).}

In its pure version, the EH implies that bond yields are fully determined by the expected path of the short-term interest rate with zero term premium. The extended version of the EH allows for a maturity-specific constant term premium. This version forms the basis of recent latent factor, semi-structural or structural models of the yield curve.\footnote{See, for example, Bekaert et al. (2010), De Graeve et al. (2009), Dewachter and Lyrio (2008), Hoedt et al. (2008), and Vasicek (1977).} If, however, bond yields consist in part of time-varying term premia not only does the EH not hold, and therefore should not be assumed in yield curve models, but also the information content of the yield curve with respect to macroeconomic aggregates may be affected. Therefore, determining the contribution of the expectations and term premium components in bond yields might allow a more precise interpretation of the dynamics of the term structure of interest rates and the construction of better information variables for macroeconomic forecasting.
The identification of the expectations and term premium components of the yield curve is, however, not an easy task. Despite the fact that the expectations theory has been rejected in a number of empirical studies, Swanson (2007) and Rudebusch et al. (2007) show that term premium estimates can differ by more than four percentage points depending on the model used. This lack of identification of term premia is not surprising given the prominent role of unobserved long-run interest rate expectations in the expectations component of the yield curve and the difficulty to identify bond risk premium dynamics within the context of a macro-finance (MF) model.

In this paper, we investigate the failure of the EH and its consequences for macroeconomic forecasting. We adopt the Extended Macro-Finance (EMF) model of Dewachter and Iania (2010), which augments standard MF models of the term structure of interest rate with the inclusion of three financial factors and two stochastic trends. The first two financial factors reflect financial strains in the money market, while the third financial factor captures time variation in bond risk premia. The two stochastic trends allow for highly persistent processes capturing time variation in long-run inflation expectations and in the equilibrium real rate, two key components of long-run interest rate expectations. We analyze, through the lens of this MF model, two relevant issues related to the failure of the EH: the dynamics of bond risk premia and the information content of the term spread and its expectations and term premium components for the forecasting of economic activity and inflation.

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To our knowledge, Hamilton and Kim (2002) were the first to decompose yield spreads into an expectations and a term premium component to forecast GDP growth. Ang et al. (2006) and Favero et al. (2005) adopt the same approach while Rudebusch et al. (2007) assess the implications of structural and reduced-form models for the relationship between term premium and economic activity. Since each of these studies adopt a different technique to decompose yield spreads, they reach different conclusions regarding the importance of each component in the forecasting of output growth. We are not aware of any study that uses such decomposition to forecast inflation.

Our analysis contributes to the current MF literature in several ways. First, we show that the EMF model is able to extract reasonable estimates for the term premium dynamics. The dynamics of our term premium is similar to Kim and Wright (2005)’s one, which is considered by Rudebusch et al. (2007) as the most representative measure among five measures examined by these authors. This is achieved by the use of a single factor as the main driving force behind bond risk premia and which captures most of the comovement in realized excess returns. This factor is similar to the return-forecasting factor proposed by Cochrane and Piazzesi (2005), the CP factor, which is however extracted by means of ad-hoc predictive regressions. When compared to realized excess returns, our model-implied risk premia are unbiased and generate an in-sample fit and out-of-sample forecasts that are in line with those generated by Cochrane and Piazzesi (2005)’s method.

Second, we find that (i) the expectations component of short-term bonds is mainly driven by
monetary policy shocks while that of long-term bonds is affected by all macro shocks and in particular long-run inflation shocks, and that (ii) movements in the term premium component are mainly associated with financial shocks, with some limited impact of liquidity and policy rate shocks. These results show that the relevance of introducing stochastic endpoints and risk premia dynamics in a MF model is not limited to the improvement of the yield curve fit (as shown in [Dewachter and Iania (2010)]), but is also essential in the identification of bond yields’ expectations and term premium components.

Third, we show that while the decomposition of the term spread into its expectations and term premium components is crucial for forecasting inflation changes, it is less relevant in the forecasting of real activity. Our results suggest that looking at the term spread to infer future changes in inflation (via, for example, the Fisher hypothesis) might be suboptimal since the information content of the term spread is affected by the presence of a risk premium component. This finding has remained stable during the past decade and is robust to the inclusion of control variables. For real activity measures like real GDP growth and the output gap, the decomposition of the term spread is less important. For GDP growth, we find that the increase in the adjusted $R^2$ is most of the time not statistically significant. For the output gap, the results are mixed. When the term spread is used as the only predictor variable, its decomposition in expectations and term premium components clearly increases the predictive content for the output gap. However, this improvement vanishes once we control for the current value of the output gap and the short-term interest rate. Furthermore, we document that during the last decade the decomposition of the term spread has lost forecasting power for both measures of real activity.
The remainder of the paper is organized as follows. Section 2 explains briefly the EMF model and discusses the implied decomposition of the yield curve in expectations and term premium components. Section 3 describes the data and the Bayesian model specification used to estimate the EMF model. Section 4 analyses the model-implied risk premia and focus on the yield decomposition and its impact in the forecasting of real activity measures and inflation. The main findings are summarized in the conclusion.

2 Affine models for bond and term premia

2.1 Bond and term premia

A standard decomposition of the default-free yield curve separates the expectations and the term premium (or yield risk premium) components. Applied to zero-coupon bonds, this decomposition takes the following form:

\[ y_t^{(n)} = \frac{1}{n} \sum_{\tau=0}^{n-1} E_t \left[ y_{t+\tau}^{(1)} \right] + \chi_t^{(n)}, \tag{1} \]

where \( y_t^{(n)} \) denotes the yield on a \( n \)-period bond at time \( t \). Equation (1) identifies the expectations component as the average expected one-period interest rate over the maturity of the bond and the term premium component as the additional compensation to lock in the money over \( n \) periods instead of rolling over \( n - 1 \) times an investment in a one-period bond.\footnote{The term premium and term spread should not be confused. The term spread refers to the difference between long- and short-run yields while the term premium measures the deviation of long-run yields from the average expected future short-term rate.}
The interpretation of the term premium can be reformulated in terms of one-period bond premia (see Ludvigson and Ng (2009)). Denote the one-period excess log return for a bond with maturity $n$ as:

$$rx_{t,t+1}^{(n)} = \ln (P_{t+1}^{(n-1)} / P_t^{(n)}) - y_t^{(1)}.$$  (2)

The term premium consists of the average one-period bond premia (or risk premia) obtained from holding the bond to maturity:

$$\chi_t^{(n)} = \frac{1}{n} \sum_{\tau=0}^{n-1} E_t [rx_{t+\tau,t+\tau+1}^{(n-\tau)}].$$  (3)

Combining equations (1) and (3), we obtain the final yield curve decomposition in terms of the expected future path of the risk-free interest rate ($E_t [y_t^{(1)}], \tau = 0, ..., n - 1$) and the expected future path of the one-period risk premium ($E_t [rx_{t+\tau,t+\tau+1}^{(n-\tau)}], \tau = 0, ..., n - 1$):

$$y_t^{(n)} = \frac{1}{n} \sum_{\tau=0}^{n-1} E_t [y_t^{(1)}] + \frac{1}{n} \sum_{\tau=0}^{n-1} E_t [rx_{t+\tau,t+\tau+1}^{(n-\tau)}].$$  (4)

Equation (4) encompasses both versions of the EH. Under the pure version, the yield on a $n$-maturity bond is fully determined by the expected path of the short-term interest rate with zero one-period risk premium at any maturity. Under the extended EH, the one-period bond premium is constant and maturity specific, i.e. $E_t [rx_{t+\tau,t+\tau+1}^{(n-\tau)}] = \phi(n)$, and all variation in the yield curve is generated by changes in market expectations about future short rates. A failure of the EH implies that the yield curve responds to changes in both the expected short-term rates and the term premia.
We model the dynamics of short rates by a vector error correction model (VECM) within a macro-finance framework. The use of a VECM instead of a standard vector autoregressive (VAR) model reflects the presence of stochastic trends driving the low frequency dynamics of the yield curve: the long-run inflation expectation and the expected equilibrium real rate (see Kozicki and Tinsley (2001) and Dewachter and Lyrio (2006)). The dynamics of one-period risk premium is driven by one specific latent factor, which we denote the risk premium or return-forecasting factor. This factor is empirically grounded in the results of Cochrane and Piazzesi (2005) and explicitly takes into account the presence of a common factor driving realized excess returns on government bonds. In the next section, we introduce the macro-finance model used in this paper and derive the model-implied components of equation (4).

2.2 The Extended Macro-Finance model of bond and term premia

The main goal of this paper is to identify the expectations and term premium components in the yield curve within a no-arbitrage framework that incorporates macroeconomic and financial factors. We, therefore, adopt the discrete-time essentially affine EMF model from Dewachter and Iania (2010). In line with Ang and Piazzesi (2003), this model includes both observable macroeconomic variables and latent factors. All latent factors, however, have a clear economic interpretation. Two of them represent long-run expectations of macroeconomic variables while the other three correspond to financial factors. Below, we first express the yield curve components based on a general MF model. We then outline the salient properties of the EMF model.
2.2.1 Macro-finance framework

The class of essentially affine MF models is characterized by three main assumptions. First, the pricing kernel, $m_t$, is assumed to be log-normally distributed and is defined as an exponentially affine function of the risk-free rate, $i_t$, and a set of Gaussian structural shocks, $\varepsilon_{t+1}$:

$$m_{t+1} = \exp(-i_t - \frac{1}{2} \Lambda_t' \Lambda_t - \Lambda_t' \varepsilon_{t+1}), \quad \varepsilon_{t+1} \sim N(0, I).$$

(5)

where $\Lambda_t$ are the market prices of risk for the structural shocks. Second, following Duffee (2002), the risk-free interest rate and the prices of risk are restricted to be affine functions of the factors, $X_t$:

$$i_t = \delta_0 + \delta_1' X_t,$$

$$\Lambda_t = \Lambda_0 + \Lambda_1 X_t.$$  

(6)

Third, the law of motion of the state vector under the historical probability measure follows a first-order VECM transition equation:

$$X_{t+1} = C + \Phi X_t + \Sigma \varepsilon_{t+1}.$$  

(7)

Based on the assumption of no-arbitrage, we can compute the price of a $n$-period bond at time $t$ by solving the following relation recursively:

$$P_t^{(n)} = E_t \left[ m_{t+1} P_t^{(n-1)} \right],$$  

(8)
with the initial condition \( P_t^{(0)} = 1 \). The resulting yields are linear functions of the state vector

\[
y^{(n)}(t) = -\frac{\ln P^{(n)}_t}{n},
\]

\[
y^{(n)}(t) = A_{y,n} + B_{y,n}X_t,
\]

where \( A_{y,n} = -a_{y,n}/n \) and \( B_{y,n} = -b_{y,n}/n \), with \( a_{y,n} \) and \( b_{y,n} \) satisfying the following no-arbitrage difference equations:

\[
a_{y,n} = a_{y,n-1} + b_{y,n-1}(C - \Sigma \Lambda_0) + \frac{1}{2} b_{y,n-1} \Sigma \Sigma' b'_{y,n-1} - \delta_0,
\]

\[
b_{y,n} = b_{y,n-1}(\Phi - \Sigma \Lambda_1) - \delta_1',
\]

given \( a_{y,0} = 0 \) and \( b_{y,0} = [0, ..., 0] \).

The model summarized by equations (7) and (10) allows an affine representation of the expected average future short rate and term premium components of the \( n \)-period yield in equation (11) and the expected excess returns in equation (3). The expectations component can be written as:

\[
\frac{1}{n} \sum_{\tau=0}^{n-1} E_t \left[ y^{(1)}_{t+\tau} \right] = \frac{1}{n} \sum_{\tau=0}^{n-1} [A_{y,1} + B_{y,1}E_tX_{t+\tau}] = A_{e,n} + B_{e,n}X_t
\]

where \( A_{e,n} = -a_{e,n}/n \) and \( B_{e,n} = -b_{e,n}/n \), with \( a_{e,n} \) and \( b_{e,n} \) determined by the following difference equations:

\[
a_{e,n} = a_{e,n-1} + b_{e,n-1}C + \frac{1}{2} b_{e,n-1} \Sigma \Sigma' b'_{e,n-1} - \delta_0,
\]

\[
b_{e,n} = b_{e,n-1}\Phi - \delta_1'.
\]
given the initial conditions $a_{e,0} = 0$ and $b_{e,0} = [0, ..., 0]$. These equations are based on the pure version of the EH, i.e. $\Lambda_0 = 0$ and $\Lambda_1 = 0$. The term premium implied by the above framework can be obtained directly from the affine representation for the yield curve and the expectations component:

$$\chi_t^{(n)} = A_{y,n} - A_{e,n} + (B_{y,n} - B_{e,n}) X_t. \quad (14)$$

Finally, the one-period expected excess return can be derived combining equations (2), (9) and (10):

$$E_t[r_{x,t,t+1}] = E_t \left[ -((n - 1))^{(n-1)} + n y_t^{(n)} - y_t^{(1)} \right] = A_{rx,n} + B_{rx,n} X_t, \quad (15)$$

where

$$A_{rx,n} = b_{y,n-1} \Sigma \Lambda_0 - \frac{1}{2}b_{y,n-1} \Sigma \Sigma' b_{y,n-1}', \quad (16)$$

$$B_{rx,n} = b_{y,n-1} \Sigma \Lambda_1.$$

Next, we specify each factor included in the EMF model, the dynamics of the state vector, and the implications for the yield curve decomposition in equations (12) and (14), and for the expected excess return in equation (15).

### 2.2.2 The Extended Macro-Finance model

The EMF model incorporates eight state variables which can be sorted in three groups. The first group includes three observable macroeconomic factors (inflation, $\pi_t$, the output gap, $y_t$, and the central bank policy rate, $i_{cb}^t$). The second group consists of three latent financial factors. The first two are related to the overall liquidity and counterparty risk in the money market...
(\(l_{1,t}\) and \(l_{2,t}\), respectively) while the third (\(l_{3,t}\)) drives the one-period bond premia. The third group contains two stochastic trends modelling the long-run equilibrium of inflation, \(\pi_t\), and the equilibrium real rate, \(\rho_t\). The state vector \(X_t\) introduced in equation (7) is, therefore, given by:

\[
X_t = [\pi_t, y_t, i_t^{cb}, l_{1,t}, l_{2,t}, l_{3,t}, \pi_t^*, \rho_t]' .
\]  

(17)

The inclusion of the three observable macroeconomic variables is standard in MF models. The introduction of liquidity factors is motivated by the recent evidence documenting the impact of liquidity shocks on the yield curve (see Christensen et al. (2009); Feldhütter and Lando (2008); Liu et al. (2006)). Moreover, the financial crisis of 2007-2008 demonstrated the significance of liquidity shocks on financial markets and the macroeconomy as a whole. In the EMF model, the liquidity factors are linked to tensions in the money market. One important measure of these money market tensions is the TED spread, i.e. the spread between the unsecured money market rate, \(i_t^{mm}\), and the Treasury bill (T-bill) rate, \(y_t^{(1)}\). This spread is considered a standard measure of funding liquidity in the money market. The two liquidity factors \((l_{1,t}, l_{2,t})\) decompose this TED spread into two components, each measuring a specific dimension of liquidity risk. The first spread factor \((l_{1,t})\) represents a convenience yield from holding Treasury bonds and can be seen as a flight-to-quality component. A flight to quality (i.e. to government bonds) is typically followed by an increase in the convenience yield, which means a widening of the spread between the yield on secured or collateralized money market rate, \(i_t^{repo}\), and the T-bill rate. The second spread factor \((l_{2,t})\) reflects a credit component and measures counterparty risk as it is given by
the difference between the unsecured and the secured money market rate. Formally:

\[ TED_t = i_t^{mm} - y_t^{(1)} = l_{1,t} + l_{2,t}, \]

\[ l_{1,t} = i_t^{repo} - y_t^{(1)}, \]

\[ l_{2,t} = i_t^{mm} - i_t^{repo}. \]

The third financial factor \((l_{3,t})\) is motivated by evidence from Cochrane and Piazzesi (2005, 2009) and Joslin et al. (2009) showing that a large fraction of the variation in bond risk premia cannot be explained by macroeconomic factors but should be modelled with an additional return-forecasting factor. This factor should capture the time variation in the one-period bond premium and therefore measures the risk attitude in the market. In the EMF framework, this factor is identified by restrictions on the prices of risk such that it accounts for all the variation in the one-period bond premium:

\[ E_t [r_{x_{lt},t+1}] = A_{rx,n} + B_{rx,n} \epsilon_6 l_{3,t} \]

where \(\epsilon_i\) is a column vector of zeros with a one on the \(i\)-th row and \(A_{rx,n}\) and \(B_{rx,n}\) are defined in equation (16).

We now turn to the third group of state variables. A number of recent papers have suggested modelling the yield curve dynamics as a cointegrated or near-cointegrated system. Dewachter and Lyrio (2006) and Dewachter and Iania (2010) include two exogenous stochastic trends in

\(^5\)These restrictions are the following:

\[ A_1(i,j) = 0, \quad \forall j \neq 6 \text{ and } \forall i. \]
the state vector leading to a cointegrated system. [Cochrane and Piazzesi (2009)] argue for a "very persistent real transition matrix", while [Joslin et al. (2009)] favor stochastic trends based on a set of formal unit root and cointegration tests. The EMF model introduces two stochastic trends, $\pi_t^*$ and $\rho_t$, representing the long-run equilibrium inflation rate and the equilibrium real rate, respectively. This macroeconomic interpretation is imposed by the following cointegrating restrictions:

$$\lim_{s \to \infty} E_t [\pi_{t+s}] = \pi_t^*, \quad \lim_{s \to \infty} E_t [\pi_{t+s}^b] = \rho_t + \pi_t^*.$$  

(20)

The introduction of stochastic trends alters the model dynamics significantly. Unlike standard MF models with fixed equilibrium levels for inflation and interest rates, the EMF model allows for time variation in the long-run expectations of these variables. This may alter considerably the model-implied expectations and term premium components of the yield curve, especially at the long end of the maturity spectrum. Importantly, in the measurement equation detailed below, we use survey data on long-horizon inflation forecasts to identify $\pi_t^*$, while the growth rate of potential real GDP is used to identify $\rho_t$.

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*Details on the identification restrictions for the stochastic trends can be found in Dewachter and Iania (2010).*
3 Estimation methodology

3.1 Data

We estimate the EMF model on U.S. quarterly data over the period 1960:Q1-2008:Q4 (196 observations). The variables included in the sample can be divided in four groups: (i) standard macroeconomic series; (ii) yield curve data; (iii) money market spreads; and (iv) a final group including survey data on inflation forecasts and potential output growth used to identify the two stochastic trends in the model. The first group of variables contains annualized inflation based on the quarterly growth of the GDP deflator, the output gap constructed from data provided by the Congressional Budget Office (CBO), and the central bank policy rate represented by the effective federal funds rate. The data are obtained from the Federal Reserve Bank of St. Louis FRED database. The second group includes per annum zero-coupon yield data for maturities of 1, 4, 8, 12, 16, 20, and 40 quarters from the Fama-Bliss Center for Research in Security Prices (CRSP) bond files with the exception of 40-quarter yields obtained from Gürgaynak et al. (2007). The third group consists of money market rates used to identify the decomposition of the TED spread into a convenience yield \((l_{1,t})\) and a credit-crunch or counterparty risk factor \((l_{2,t})\). We use the 1-quarter Eurodollar (Ed) rate and the 1-quarter London Interbank offered rate - LIBOR (Lb) from Datastream to identify the credit-crunch factor. We supplement the LIBOR data with data on the Eurodollar given that the latter series dates back further in time. Furthermore, we use the 1-quarter T-bill rate to identify the convenience yield. All spreads are computed

\footnote{Given that the Eurodollar and the LIBOR rates are closely related, we use the former as an additional proxy for the credit-crunch factor.}
relative to the secured money market rate represented by the government-backed collateral repo.
rate (GC-repo) from Bloomberg (ticker RPGT03M). The fourth group includes survey data
on the average 4- and 40-quarter inflation forecasts and data on potential output growth. The
data on inflation forecasts are retrieved from the Survey of Professional Forecasters (Federal
Reserve Bank of Philadelphia) and are used to identify long-run inflation expectations. The
data on potential output growth, measured as the quarterly growth of CBO potential output,
are included to identify the equilibrium real rate.

In the forecasting exercise of Section 4.2.2 we use one additional dataset. It comprises inflation
forecasts from the Greenbook dataset provided by the Federal Reserve Bank of Philadelphia for
the period 1974:Q2 to 2005:Q4 (127 obs.). The data consist of annualized quarterly growth rate
of the GDP deflator and end in 2005 due to the five-year lag between the forecast and the release
date. The forecasts are made at the middle of the quarter for the current quarter and for one
to four quarters ahead.

### 3.2 Econometric setting

The EMF model contains a total of 92 parameters represented by the vector $\theta$. We estimate the
model using standard Bayesian techniques based on informative priors. We use a large number
of observable variables over a long time span to generate sufficient degrees of freedom in the
estimation. The posterior density of $\theta$, $p(\theta \mid Z^T)$, can be written as:

$$p(\theta \mid Z^T) = \frac{L(\theta \mid Z^T)p(\theta)}{p(Z^T)},$$  (21)
where $Z^T$ denotes the dataset, $L(\theta \mid Z^T)$ the likelihood function, $p(\theta)$ the priors, and $p(Z^T)$ the marginal density of the data. Following Smets and Wouters (2007), among others, we use a two-step procedure to simulate the posterior density of the parameters. In a first step, we find the mode of the posterior distribution of $\theta$ using a combination of Newton-Raphson and simulated annealing optimization procedures. Subsequently, we use the random walk Metropolis-Hastings procedure to trace the posterior density of $\theta$. Given the large number of parameters, we use a large amount of draws and check convergence by means of the standard battery of convergence tests.

Next, we present the likelihood function and the set of priors used to estimate the model.

### 3.2.1 The likelihood function

The likelihood function is obtained from the prediction error decomposition implied by the measurement equation. We rewrite the state space dynamics in equation (7) making explicit the dependence on the parameter vector $\theta$:

$$X_{t+1} = C(\theta) + \Phi(\theta)X_t + \Sigma(\theta)\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I). \quad (22)$$

The measurement equation relates the observed data $Z_t$ to the state vector $X_t$:

$$Z_t = A(\theta) + B(\theta)X_t + S(\theta)\eta_t, \quad \eta_t \sim N(0, I). \quad (23)$$

As mentioned before, we use four groups of information variables in the measurement equation. The observed series in $Z_t$ consist of (i) macroeconomic variables ($Z_{mac,t}$), (ii) yield curve data

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*The method used is similar to the one discussed in Dewachter and Iania (2010).*
(Z_{g,t}), (iii) money market spreads (Z_{mm,t}), and (iv) data used to identify the two long-run trends in the model (Z_{LR,t}):}

\[ Z_t = [Z'_{mac,t}, Z'_{y,t}, Z'_{mm,t}, Z'_{LR,t}]', \quad (24) \]

where

\[ Z_{mac,t} = [\pi_t, y_t, i^{cb}_t]', \quad Z_{y,t} = [y_t^{(1)}, y_t^{(4)}, y_t^{(8)}, y_t^{(12)}, y_t^{(16)}, y_t^{(20)}, y_t^{(40)}]', \quad (25) \]

\[ Z_{mm,t} = [i^{Lb}_t - y_t^{(1)}, i^{Ed}_t - y_t^{(1)}, i^{GC}_t - y_t^{(1)}]', \quad Z_{LR,t} = [F_{\pi,t}^{(4)}, F_{\pi,t}^{(40)}, \Delta y_t^{pot}]', \]

The vector of constants \( A(\theta) \), the matrix of factor loadings \( B(\theta) \), and the matrix \( S(\theta) \) are accordingly partitioned in four blocks:

\[ A(\theta) = [A'_{mac}, A'_y, A'_{mm}, A'_{LR}]', \]

\[ B(\theta) = [B'_{mac}, B'_y, B'_{mm}, B'_{LR}]', \quad (26) \]

\[ S(\theta) = diag(S'_{mac}, S'_y, S'_{mm}, S'_{LR}). \]

A number of observations can be made. First, we assume that the three macroeconomic variables \((\pi_t, y_t, i^{cb}_t)\) are observed without errors, implying that \( A_{mac} = 0_{3 \times 1} \), \( B_{mac} = [I_{3 \times 3}, 0_{3 \times 5}] \), and \( S_{mac} = 0_{3 \times 1} \). Second, all yields are measured with an error and are related to the state variables through the no-arbitrage equation \((10)\). Third, we use three money market spreads to identify the convenience yield and credit-crunch factors \((l_{1,t} \text{ and } l_{2,t}, \text{ respectively})\). We use two measures
for the TED spread since LIBOR rates are only available from 1986:Q2 onwards. For the period 1971:Q2-1986:Q1, we use the TED spread based on the Eurodollar rate \((i_t^{Ed} - y_t^{(1)})\). After that, the TED spread is based on the LIBOR rate \((i_t^{Lib} - y_t^{(1)})\). Both are used to identify the credit-crunch factor \((l_{2,t})\). We assume there is a spread between the Eurodollar and LIBOR rates equal to a constant, \(c_{Ed}\), plus an idiosyncratic shock, \(\eta_{Ed,t}\):

\[
i_t^{Ed} = y_t^{(1)} + c_{Ed} + l_{1,t} + l_{2,t} + \sigma_{Ed} \eta_{Ed,t}.
\]

(27)

The third spread is based on the GC-repo rate \((i_t^{GC} - y_t^{(1)})\) and identifies the convenience yield \((l_{1,t})\) perfectly. The above identification implies that

\[
A_{mm} = \begin{bmatrix}
0 \\
c_{Ed} \\
0
\end{bmatrix}, \quad B_{mm} = \begin{bmatrix}
0_{1 \times 3} & 1 & 1 \\
0_{1 \times 3} & 1 & 0_{1 \times 3} \\
0_{1 \times 3} & 0 & 0_{1 \times 3}
\end{bmatrix}, \quad S_{mm} = \begin{bmatrix}
0 \\
\sigma_{Ed} \\
0
\end{bmatrix}.
\]

(28)

Finally, survey data on 4- and 40-quarter average inflation forecasts \(F^{(4)}_{\pi,t}\) and \(F^{(40)}_{\pi,t}\) are used to identify the stochastic trend for inflation. The loadings for these survey expectations are implied by the transition equation (7). The stochastic trend for the real rate is identified by the growth rate of potential output:

\[
\Delta y_t^{pot} = \alpha + \beta_{\rho} \rho_t + \sigma_{\Delta y^{pot}} \eta_{\Delta y^{pot},t}
\]

(29)

where \(y_t^{pot}\) denotes log potential output. We allow for measurement errors in each of the series:

\[S_{LR} = [\sigma_{F^{(4)}_{\pi,t}}, \sigma_{F^{(40)}_{\pi,t}}, \sigma_{\Delta y^{pot}}]^T.\]

\[\text{The LIBOR rate is an average of rates at which banks offer funds (offer side), while the Eurodollar rate refers to a rate at which banks want to borrow funds (bid side). Typically, the Eurodollar rate is about one basis point below the LIBOR rate.}\]
The log-likelihood function is obtained by exploiting the linearity and normality of the system composed by equations (22) and (23):

\[
    l(Z_t|\theta) = -\frac{T}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \left[ \ln(\det(V_{Z,t|t})) + (Z_t - Z_{t|t-1})' (V_{Z,t|t})^{-1} (Z_t - Z_{t|t-1}) \right]
\]

(30)

with the prediction error, \(Z_t - Z_{t|t-1}\), and its variance, \(V_{Z,t|t}\), given by Kalman filter recursions (see Harvey (1991)).

3.2.2 Prior distributions

Overall, we use relatively loose priors characterized by large standard deviations of the prior distributions. Most of the priors reflect standard beliefs regarding macroeconomic dynamics. The priors incorporate significant inertia in the dynamics of macroeconomic variables and impose a delayed deflationary impact of changes in the policy rate. Also, the priors for the dynamics of the policy rate reflect a Taylor-rule type of monetary policy. The prior distributions on the impact matrix \(\Sigma\) identify a supply, a demand and a policy rate shock, while modelling the financial shocks as demand shocks affecting inflation and the output gap negatively. Finally, the priors for the prices of risk are constructed such that the EMF model implies an upward-sloping yield curve (see also Chib and Ergashev (2009)).

\(^{10}\)Table 1 in the appendix lists the type of distribution, mean and standard deviation for the prior of the parameter vector \(\theta\).
4 Empirical results

In this section, we discuss the implications of the estimated EMF model and the implied decomposition of the yield curve for the prediction of excess bond returns, real economic activity, and inflation. In section 4.1 we assess the time variation in the model-implied bond premia and the predictive power of the EMF model to forecast excess returns. We find significant time variation in bond premia (in line with Cochrane and Piazzesi (2005, 2009)) suggesting a rejection of the EH, and find that the extracted return-forecasting factor is closely related to the CP factor in terms of forecasting power of realized excess returns. In section 4.2 we first examine the importance of the expectations and term premium components for the time variation in long-term yields. This allows us to assess the extent to which changes in long-maturity yields reflect shifts in expectations about future short-term rates or in term premia. Finally, we assess the impact of this decomposition to predict real economic activity and inflation.

4.1 Bond risk premia

The EMF model clearly rejects the extended EH. The model generates significant time variation in the risk premia capturing a significant part of the variability of realized excess returns. These risk premia are driven by a common factor that is closely related to the return-forecasting factor of Cochrane and Piazzesi (2005).

Figure 1 illustrates the time variation in bond premia implied by the EMF model. This figure

11The parameter estimates of the EMF model are presented in Tables 2 to 4 of the appendix.
shows the expected excess return (risk premium) over a 4-quarter holding period for 8- to 20-quarter bonds against the corresponding realized excess return. The risk premia are statistically significant and display strong time variation and collinearity across maturities. The latter feature is indicative of a dominant factor in bond premia, i.e. the return-forecasting factor. Figure 2 compares the return-forecasting factor implied by the EMF model with that of Cochrane and Piazzesi (2005). The correlation between the two series is 67%. Statistical significance of the time variation in bond premia can be assessed based on the 99% error bands shown in Figure 1. Based on the error bands, the EH-implied null hypothesis of constant bond premia is rejected. More formal evidence against the EH is obtained by analyzing the posterior of the prices of risk (see Table 1). This can be seen by observing that the time-varying prices of risk related to inflation and interest rates, i.e. $\lambda_1(1,6)$ and $\lambda_1(3,6)$, are different from zero and mainly concentrated on the positive and negative sides of the support, respectively. This suggests that bond premia comove with the return-forecasting factor and that inflation and interest rate exposures are significantly priced. Finally, in line with the literature we observe that risk premia tend to be countercyclical in the EMF model. The 4-quarter expected excess return for 8- to 20-quarter bonds implied by the EFM model has a correlation of around -45% with the output gap.

A more formal comparison between the Cochrane and Piazzesi (2005) and the EMF models can be found in Table 2, where we report in-sample and out-of-sample results for excess bond premia. Note that all risk components are priced, i.e. carry at least a constant risk premium. The constant prices of risk ($\lambda_0$) are statistically significant for each of the shocks.
returns. The top three panels of this table report the adjusted $R^2$ expressing the fraction of realized excess returns explained by the EMF model and a model based on the CP factor. This is done for a 4-, 8-, and 16-quarter holding period and for bonds with maturities between 8 to 20 quarters. In general, we find that the EMF model explains a substantial amount of the variation in realized excess returns. This finding is in line with Cochrane and Piazzesi (2005, 2009) and Ludvigson and Ng (2009), who show that a limited number of factors can forecast a significant part of realized excess returns. For the 4-quarter horizon, the performance of both models is comparable, predicting on average above 30% of the in-sample variation in the realized excess returns. For the 8-quarter horizon, while the CP factor explains on average 21% of the variability in realized excess returns, the EMF factor explains on average almost 40%.

Insert Table 2

As an additional test, we check the unbiasedness of the EMF model-implied bond risk premia. To this end, we regress realized excess returns on expected excess returns as implied by the EMF model:

$$\text{rx}^{(n)}_{t,t+k} = \alpha + \beta E_t \left[ \text{rx}^{(n)}_{t,t+k} \right] + \varepsilon_{t+k}, \quad n = 8, 12, 16, 20 \text{ qtr}, \quad k = 4, 8, 16 \text{ qtr}, \quad n > k,$$

where $n$ denotes the maturity of the bond, $k$ represents the holding period, $\text{rx}^{(n)}_{t,t+k}$ denotes the realized return in excess of the $k$-quarter risk-free rate of buying a $n$-quarter bond at time $t$ and selling it back after $k$ quarters, and $E_t \left[ \text{rx}^{(n)}_{t,t+k} \right]$ represents the EMF model-implied risk
premium on a $n$-quarter bond over a $k$-quarter period. To validate the EMF model, we test the joint hypothesis that $\alpha = 0$ and $\beta = 1$. Table 3 shows that the risk premia implied by the EMF model are unbiased: (i) all $\alpha$ coefficients are statistically insignificant while the $\beta$ coefficients are not statistically different from one, and (ii) based on a standard $F$-test, we cannot reject the joint hypothesis that $\alpha = 0$ and $\beta = 1$ at any significance level for any regression.

Insert Table 3

Next to performing an in-sample analysis, we also perform an out-of-sample analysis for the 4- and 8-quarter excess return over the period 1996:Q1-2008:Q4. We compare the performance in terms of the mean square error (MSE) of the EMF model against the CP factor and a random walk model with drift (i.e. with constant risk premium). The bottom two panels of Table 2 show that for the 4-quarter horizon, the EMF model has a slightly superior performance against the other two models, except against the random walk model for a 20-quarter horizon. For the 8-quarter horizon, the EFM model outperforms all other models in forecasting excess returns for all maturity bonds. Notwithstanding the finding that the EMF model outperforms the alternative models, it is also necessary to acknowledge the significant decrease in the predictive power of all models in out-of-sample exercises.

An implication of the EMF model is that bond premia are mainly driven by financial shocks; macroeconomic shocks, in contrast, only contribute marginally to the dynamics of the risk premia. Panel (a) of Table 4 illustrates the relevance of financial shocks for risk premia by
means of a variance decomposition of the 4-quarter bond premia of 8- and 20-quarter maturity bonds. The results highlight the importance of three types of shocks for bond premium dynamics: return-forecasting factor shocks (i.e. risk premium shocks), liquidity shocks, and policy rate shocks. The return-forecasting factor shocks are the dominant source of variation in bond premium. Depending on the prediction horizon, this type of shock explains between 60% and 80% of the total variation in bond premia. Liquidity shocks explain between 12% and 20% of the bond premium variation. Finally, for long horizons, we observe a significant role for monetary policy shocks, i.e. explaining around 15% of the variation in bond premia.

Insert Table 4

4.2 Term premia

The rejection of the EH raises the question of the relative importance of the expectations and term premium components in the yield curve dynamics. This issue is relevant since yield curve changes might have different macroeconomic interpretations depending on the source of its variation (see Rudebusch et al. (2007) and Ludvigson and Ng (2009) for a detailed treatment of the topic), in particular when one is considering the time variation in long-term yields. In this

13The ordering of the variables is the same as the one in the state vector (Eq. 17).
14Note that, by construction, shocks to the return-forecasting factor explain fully the variation in the one-quarter holding period return. Since we analyze one-year holding period returns, other factors (shocks) may impact the bond premia through their impact on the dynamics of the return-forecasting factor.
15A decrease in long-term yields generated by a decrease in the expectations component or term premia leads to different interpretations. Decreases in risk premia will be stimulating and hence may call for restrictive monetary measures. This point has been stressed by Bernanke (2006): "...when the term premium declines, a higher short-term rate is required to obtain the long-term rate and the overall mix of financial conditions consistent with maximum sustainable employment and stable prices". On the contrary, decreases in the expectations component typically signal expectations about a future economic slowdown and would call for more expansionary measures.
section, we first decompose bond yields in an expectations and a term premium part and study
the macroeconomic and financial drivers of these two components. Subsequently, we assess the
predictive power of a popular yield curve indicator, the term spread. We study the predictive
downloading power of such indicator and of its expectations and term premium components with respect to
real economic activity and inflation.

4.2.1 Decomposing the yield curve

The decomposition of the yield curve into expectations and term premium components is defined
in equation [1]. We illustrate this decomposition in Figure 3. The top panel of this figure
displays the fitted time series of the 40-quarter yield and the middle panel plots the expectations
component of this yield. The bottom panel displays the term premium implied by the EMF
model and compares it to the Kim and Wright (2005) measure (KW) [16]. According to Rudebusch
et al. (2007), among five measures of term premia considered by these authors the KW measure
seems to be the most representative of them (see also Rosenberg and Maurer (2008)). We note
that, despite the significant differences in structure between the EMF and KW models [17], the
term premia derived from these models are remarkably similar. This result might be surprising,
especially in light of the findings of Rudebusch et al. (2007). They compare different measures of
the term premium and find that the behavior of the KW and the Bernanke et al. (2004) measures
are remarkably similar while that of Cochrane and Piazzesi (2005) is harder to understand since

16 The Federal Reserve Board provides data to generate the term premium from the Kim and Wright (2005)
model.
17 The Kim and Wright (2005) model is a standard latent factor model augmented with survey data, whereas
the EMF model combines macroeconomic, yield curve, and survey data.
it is well below the other measures and is far too volatile. Our EMF model is able to filter a return-forecasting factor similar to the CP factor while generating a term premium measure similar to that of Kim and Wright (2005).

Insert Figure 3

The time variation in our term premium series is substantial, which indicates that the rejection of the EH documented above has significant economic implications. In particular, the one-to-one relation between yields and expected short rates (implying a constant, maturity-specific term premium) breaks down, especially for long-term bonds.

Panels (b) and (c) of Table 4 show the variance decomposition of the expectations and term premium components, respectively, for bonds with maturities of 4 and 40 quarters. The expectations component of 4-quarter bonds is dominated by monetary policy shocks while of long-term (40-quarter) bonds is dominated by long-run inflation shocks. In line with the findings of the previous section, the term premium component is driven mainly by risk premium shocks both for short- and long-term bonds. Liquidity and policy rate shocks have a smaller effect over all horizons while macroeconomic shocks are insignificant. Therefore, the substantial time variation in the term premia and the fact that this variation is primarily linked to financial and not macroeconomic shocks may reduce the informational content of the yield curve with respect to future macroeconomic developments. As a consequence, it may also blur the informational content of yield spreads which is the measure mostly used for macroeconomic predictions.
Yield spreads can also be decomposed into expectations and term premium components and each of these components aggregates both macroeconomic and financial shocks with their relative importance changing over time. Therefore, by decomposing yield spreads we obtain information at a more disaggregated level, allowing for a better identification of the shocks. Figure 4 illustrates this decomposition. The top panel of this figure shows the 40-quarter yield spread implied by the EMF model and the middle and bottom panels display its expectations and term premium components, respectively. This figure suggests that a significant part of the yield spread variation is due to the time variation in the term premium, making it less clear the informational content of yield spreads with respect to future macroeconomic variables.

Insert Figure 4

In the next section, we assess the predictive content of yield spreads and their decomposition for macroeconomic predictions.

4.2.2 Macroeconomic information in the yield curve

We assess the information content of the EMF model-implied expectations ($Spr_{t}^{e,(n)}$) and term premium ($\chi_{t}^{(n)}$) components of yield spreads ($Spr_{t}^{(n)}$) in the predictive regressions of real economic activity and inflation, with $Spr_{t}^{(n)} = y_{t}^{(n)} - y_{t}^{(1)} = Spr_{t}^{e,(n)} + \chi_{t}^{(n)}$. We concentrate on two measures for economic activity: real GDP growth and output gap. Our analysis on GDP growth is closely related to [Ang et al. 2006], [Estrella and Mishkin 1997] and [Rudebusch et al. 2007], while the prediction exercise for output gap is relatively new. We also use two measures for our
analysis on inflation. We forecast inflation taking into consideration the main results of Faust and Wright (2011), and predict forward inflation changes as in Estrella and Mishkin (1997) and Mishkin (1990).

**Predicting economic activity** We follow Ang et al. (2006) in assessing the information content of the yield curve for GDP growth. We estimate several predictive regressions where the most extended version regresses the cumulative log real GDP growth for the next $k$ quarters on the yield spread components:

$$g_{t-k} = \alpha + \beta^{EC} (Spr_{t}^{e,(n)} + \chi_{t}^{(n)}) + \beta^{TP} \chi_{t}^{(n)} + \gamma g_{t} + \delta y_{t}^{(1)} + \varepsilon_{t+k}$$ (32)

where $g_{t} \equiv g_{t-1-t}$ denotes GDP growth in the past quarter, all expressed in yearly terms. In line with the literature, we use lagged GDP growth, $g_{t}$, and the short-term interest rate, $y_{t}^{(1)}$, as control variables. For the output gap prediction, we use a specification similar to equation (32):

$$y_{t+k} = \alpha + \beta^{EC} (Spr_{t}^{e,(n)} + \chi_{t}^{(n)}) + \beta^{TP} \chi_{t}^{(n)} + \gamma g_{t} + \delta y_{t}^{(1)} + \varepsilon_{t+k}$$ (33)

where $y_{t+k}$ and $y_{t}$ denote the output gap at time $t+k$ and $t$, respectively. In both cases, we distinguish between four types of models. In the first two types, we assume that the yield spread is a sufficient and exhaustive information variable for output growth (output gap), i.e. $\gamma = \delta = 0$. This first type of model is therefore the standard representation based solely on the spread. This specification implies that the source of the spread (i.e. expectations or term premium component) is irrelevant for output growth (output gap) predictions, i.e. it imposes...
The second type of model allows for differential informational content in the spread components, i.e. it allows $\beta^{TP} \neq 0$. Note that, by construction, a statistical test for the relevance of the decomposition consists in testing for the rejection of the null hypothesis $H_0 : \beta^{TP} = 0$.

The third and fourth types allow for additional control variables, being the lagged GDP growth (current output gap) and the short-term interest rate. Similar to the first type, the third type of model does not distinguish between the sources of the spread. The last type of model is the most general one as in equations (32) and (33), differentiating between the sources of the spread and allowing for control variables. We estimate each model using either the 4-, 20-, or 40-quarter yield spread and for a GDP growth (output gap) horizon of 1, 4, and 8 quarters.

Table 5 summarizes the results from the regression analysis for GDP growth (Eq. (32)). The estimates for model 1 report the known result that the yield spread is a valuable information variable for the prediction of future GDP growth. Yield spreads are statistically significant irrespective of the prediction horizon and indicate that high yield spreads predict positive GDP growth. Despite their significant predictive content, yield spreads are not sufficient statistics for GDP growth predictions. Adding lagged GDP growth and the short-term interest rate to the regression improves the performance of the predictive equations in almost all cases if the yield spread is used (model 3) and in all cases if the decomposed spread is used (model 4). Note that the inclusion of control variables (model 3) does not drive out the yield spread as a predicting variable. In both cases (models 3 and 4), lagged GDP growth is statistically positive for horizons of 1 and 4 quarters but not for a 8-quarter horizon. This shows that previous GDP growth is important to predict future short-term growth but less relevant for longer horizons. In both
models 3 and 4, all estimates for the coefficient on the short-term interest rate have a negative sign, although only statistically significant when the 4-quarter spread is used. A negative sign indicates that the predictive content of yield spreads depends also on the level of the interest rate and should be lowered for high interest rate levels.

We now assess the impact of the yield spread decomposition to forecast GDP growth. Although a simple decomposition of the yield spread (model 2) improves forecasts of GDP growth, the increase in the adjusted $R^2$ is smaller than the one obtained with the use of control variables (model 3). Finally, comparing models 1 and 2 and models 3 and 4, we observe that although the decomposition of the yield spread into its two components leads in most cases to an increase in the adjusted $R^2$, in all cases we cannot reject the hypothesis that $\beta^{TP} = 0$. Therefore, surprisingly, the yield spread decomposition as implied by the EFM model does not seem to improve the prediction of GDP growth.

The results in the literature regarding the importance of each yield spread component are contradictory. Our results are in line with Ang et al. (2006), who find that only the expectations component is relevant for forecasting output growth. According to Hamilton and Kim (2002), however, both expectations and term premium components add explanatory power with respect to the GDP growth. Favero et al. (2005) in turn attribute more importance to the term premium component. The same authors also show that for both the Ang et al. (2006)'s and the Hamilton

\[15\]This negative relation between the interest rate level and future GDP growth is discussed in Ang et al. (2006).

\[16\]Interestingly, Ang et al. (2006) recommend for prediction purposes the use of the longest maturity yield to measure the spread. In their case, this is the 20-quarter yield. Our longest yield has a maturity of 40 quarters but we find that in 9 out of 12 cases the best spread to be used in order to forecast GDP growth is the 20-quarter spread, while the 4-quarter spread provides the best performance in the remaining 3 cases.
and Kim (2002)'s decomposition the expectations component loses its significance when one includes the short rate and inflation as control variables. Rudebusch et al. (2007), on the other hand, focus the analysis on the impact of a change in the term premium on output growth. In contrast to most of the empirical evidence, they show that a decline in the term premium is associated with an increase in output growth. This is in line with the results of our model 2, although our coefficients are not statistically significant. Furthermore, once we include control variables (model 4), we obtain in most cases the opposite result.

Insert Table 5

Table 6 presents the results from the regression analysis for the output gap (Eq. (33)). The estimates for model 1 show that the yield spread alone has some minor predictive ability for the output gap for horizons of 1 and 8 quarters and only making use of 20- or 40-quarter yield spreads. The results for model 2 indicate that a decomposition of yield spreads in expectations and term premium components improves significantly the forecasting ability of the yield spread. The expectations component signals most of the time an increase in the output gap, and in seven out of nine cases it has a significantly positive sign. The results for model 3, however, reveal that for forecasting horizons up to one year the inclusion of control variables have a greater impact on the forecasting ability for the output gap than the decomposition of the yield spread. This is mostly due to the correlation structure in the output gap series, which is particularly strong for short lags. For the two-year horizon, nevertheless, the inclusion of control variables results in an increase of the adjusted $R^2$ of 10 percentage points on average, while the
decomposition of the yield spread (model 2) leads to an equivalent increase of 8 percentage points. Therefore, for longer forecasting horizons the term spread components have for output gap an almost equivalent information content in comparison with the current level of the output gap and the short-term interest rate. Finally, the results for model 4 show, with one exception, that once control variables are included we cannot reject the hypothesis that $\beta^{TP} = 0$. As a consequence, comparing models 3 and 4 we conclude that the increase in the adjusted $R^2$ due to the decomposition of the yield spread is no longer significant. Hence, once we control for the current level of the output gap and the short-term interest rate the yield spread decomposition does not seem to contribute to the prediction of the output gap.

Insert Table 6

We analyze now whether the predictive content of the yield spread and its components has changed over time. We reestimate the EFM model at every quarter using an expanding window and use expanding windows for the predictive regressions. We concentrate on the analysis for GDP growth. The rows of panels in Figure 5 define the predictive horizon used in the regression (1, 4, and 8 quarters) and the columns of panels define the maturity of the yield spread used in the regression (4, 20, and 40 quarters). The graphs show the end date of the sample period used and the resulting adjusted $R^2$. Therefore, the first point in each graph indicates the adjusted $R^2$ for the period 1960:Q1-1995:Q4 and the last point the adjusted $R^2$ for the full sample.

Insert Figure 5
From Figure 5, we observe a general decrease in the predictive power over time of the yield spread and its components to forecast GDP growth. This decrease seems to be stronger after 2002. The figure makes it clear that a simple yield spread decomposition (i.e. without control variables) has a higher impact for short-term forecasting horizons. For the 1-quarter horizon (first row of panels) there is a significant improvement in the adjusted $R^2$ simply by decomposing the spread. This improvement is less marked for longer forecasting horizons (second and third rows of panels). The opposite happens if one allows for control variables, i.e. the yield spread decomposition becomes more important for long-horizon forecasts. For the 1- and 4-quarter horizons, there is little improvement from the term spread decomposition. This improvement is significant for the 8-quarter horizon although such gain has decreased over time.

**Predicting inflation**  We investigate the contribution of the yield spread decomposition in forecasting inflation using two sets of predictive regressions. The first set of regressions is based on the recent work of Faust and Wright (2011), who analyze seventeen methods to forecast inflation. Faust and Wright (2011) find that for our measure of inflation (GDP deflator) the Federal Reserve’s Greenbook forecast outperforms most model-based forecasts and that the random walk-based model of Atkeson and Ohanian (2001), the RW-AO, does remarkably well in forecasting inflation. The RW-AO model uses the average quarterly inflation over the past four

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20 This fact was also observed by Dotsey (1998), Favero et al. (2005), and Haubrich and Dombrosky (1996).
21 Figure 4 of the appendix shows the evolution over time of the predictive content of the yield spread and its components for the output gap. The predictive power of the yield spread and its components has remained almost constant over time with a slight decrease at the end of the sample period. Also, in general, the inclusion of control variables has a much higher impact on the adjusted $R^2$ than the decomposition of the yield spread.
22 Ang et al. (2007) and Faust and Wright (2009) show a similar result. Faust and Wright (2011) use three types of survey. The other two are the Survey of Professional Forecasters provided by the Federal Reserve Bank of Philadelphia and the Blue Chip inflation survey.
quarters as the forecast for inflation \( k \) periods ahead. Given these findings, we assess whether 

(i) the yield spread decomposition has predictive power to forecast inflation beyond the RW-AO method, and whether (ii) this forecasting power is robust to the inclusion of a set of control variables, including the Greenbook forecast.

The second set of regressions is based on the work of Estrella and Mishkin (1997) and Mishkin (1990) who examine the information in the longer maturity term structure to forecast the future path of inflation. We therefore regress the change in the future \( k \)-year inflation rate from the 1-year inflation rate on the yield spread decomposition. In both sets of regressions, we use two control variables, that is the lagged value of the dependent variable and the short-term interest rate.

Our first forecasting exercise is based on the RW-AO model. This model predicts that the inflation in \( k \) periods is equal to the average of quarterly inflation over the past four quarters,

\[
\bar{\pi}_{t-3,t} = \frac{1}{4} \sum_{j=0}^{3} \pi_{t-j}.
\]

Hence, we evaluate the predictive power of yield spreads – and their decomposition in expectations and term premium components – to predict the deviation of future inflation \( k \) periods ahead from the forecast based on the RW-AO model at time \( t \),

\[
\bar{\pi}_{t-3,t}:
\]

\[
\pi_{t+k} - \bar{\pi}_{t-3,t} = \alpha + \beta EC \left( Spr_t^{e(n)} + \chi_t^{(n)} \right) + \beta TP \chi_t^{(n)} + \gamma (\pi_t - \bar{\pi}_{t-4,t-1}) + \delta y_t^{(1)} + \varepsilon_{t+k} \tag{34}
\]

where \( \pi_{t+k} \) is the level of inflation between quarter \( t + k - 1 \) and \( t + k \), expressed in annual terms. We allow for a set of control variables, that is the deviation from current inflation and the
average of quarterly inflation over the periods $t - 4$ and $t - 1$, $\pi_t - \bar{\pi}_{t-4,t-1}$, and the short-term interest rate, $y^{(1)}_t$. We predict inflation 1, 4 and 8 quarters ahead ($k$) using 4-, 20- and 40-quarter ($n$) yield spreads. As for the predictive regressions of economic activity, we characterize four model versions: (i) incorporating the spread (models 1 and 3) or the decomposed spread (models 2 and 4); and (ii) not including control variables (models 1 and 2) or including control variables (model 3 and 4).

Table 7 shows that yield spreads alone (model 1) are statistically significant only for a horizon of 8 quarters with only minor predictive ability. Using decomposed yield spreads (model 2), however, we observe a significant increase in the adjusted $R^2$ for all forecasting horizons and spread maturities. Moreover, the $t$-statistics for $\beta^{TP}$ clearly rejects the null hypothesis $H_0 : \beta^{TP} = 0$, showing the statistical significance of the yield spread decomposition. If we allow for control variables instead of decomposing the yield spread (model 3 versus model 1), we also observe a significant increase in the adjusted $R^2$. However, a comparison between models 1 and 2 and models 1 and 3 shows that for horizons above one year the yield spread decomposition has a larger effect on the adjusted $R^2$ than the inclusion of control variables. The results of model 4 moreover show that even allowing for control variables the yield spread decomposition is still statistically significant for forecasting horizons above one year. Furthermore, the increase in the adjusted $R^2$ is higher for longer forecasting horizons. Comparing the coefficients for the yield spread ($\beta$) and its components ($\beta^{EC}$ and $\beta^{TP}$) in models 3 and 4, respectively, we observe that allowing for separate effects from the yield spread components increases the information content of the term spread, i.e. the separate coefficients increase in magnitude due to their opposite
signs. Also, while in model 3 some of the coefficients on the yield spread are negative, once you allow for the term spread decomposition all coefficients on the expectations component have a positive sign. We conclude that the yield spread decomposition is crucial for forecasting inflation and becomes more important as the forecasting horizons increases.

The regression results from equation (34) show the importance of decomposing yield spreads to forecast the deviation of future inflation from the RW-AO measure of inflation, which was praised by [Faust and Wright (2011)] based on its predictive power. We go further and include in the predictive regressions a subjective inflation forecast as one extra control variable:

\[
\pi_{t+k} - \bar{\pi}_{t-3,t} = \alpha + \theta Sur^k_t + \beta^{EC}(Spr_{t}^{e,(n)} + \chi_{t}^{(n)}) + \beta^{TP} \chi_{t}^{(n)} + \gamma (\pi_{t} - \pi_{t-1,t-4}) + \delta y_{t}^{(1)} + \varepsilon_{t+k}
\]

where \(Sur^k_t\) denotes the Greenbook forecast of inflation \(k\) quarters ahead. Table \[8\] shows the results for two models. Yield spreads are included in model 1 (i.e. \(\beta^{TP} = 0\)) and decomposed spreads are incorporated in model 2. For each forecasting horizon \((k = 1\) and \(4\) quarters) and yield spread \((n = 4, 20,\) and \(40\) quarters), we also assess the contribution of the Greenbook forecast by performing the regression (35) with and without the Greenbook data. The first column for each maturity in models 1 and 2 are equivalent to models 3 and 4 of Table \[7\]. The results are not identical due to the difference in the sample period.

The results that emerge from Table \[8\] are mixed. When we do not use decomposed term spreads,
the Greenbook forecast is important for forecasting inflation only for a one-year horizon. This might be expected since we are forecasting the deviation of future inflation from the RW-AO model, selected by Faust and Wright (2011) due to its ability to forecast inflation. Comparing models 1 and 2, the yield spread decomposition leads to an increase in the adjusted $R^2$ in 5 out of 6 cases once we control for the Greenbook forecast (second column for each maturity). The exception is the forecast for the 4-quarter horizon using a 4-quarter yield spread. And in 3 out of these 5 cases, the coefficient on the term premium component ($\beta^{TP}$) is statistically significant, stressing the importance of the term premium component. Focusing our analysis on the cases where the use of the Greenbook forecast leads to an increase in the adjusted $R^2$ (one-year forecasting horizon in model 1), we note that using a 20-quarter yield spread the spread decomposition still leads to a further increase in the adjusted $R^2$. We conclude that the yield spread decomposition implied by the EFM model adds explanatory power for forecasting horizons above one year even after the inclusion of the Greenbook forecast which is considered by Faust and Wright (2011) as one of the best predictors of inflation.

Insert Table 8

Our second set of regressions is based on the work of Estrella and Mishkin (1997) and Mishkin (1990) who show that an increase in the term spread is an indication of positive changes in future inflation. We follow Mishkin (1990) and regress forward inflation changes on yield spread
components, past inflation and the short-term interest rate:

\[ \pi_t^{(k)} - \pi_t^{(4)} = \alpha + \beta EC \left( \text{Spr}_t^{(k)} + \chi_t^{(k)} \right) + \beta TP \chi_t^{(k)} + \gamma \pi_t^{(-4)} + \delta y_t^{(1)} + \varepsilon_{t+k} \]  

(36)

where \( \pi_t^{(k)} - \pi_t^{(4)} \) is the difference between the future \( k \)-quarter inflation rate from time \( t \) to \( t + k \) expressed in annual terms and the future 4-quarter inflation rate from \( t \) to \( t + 4 \). The control variable \( \pi_t^{(-4)} \) denotes past inflation between \( t - 4 \) quarters and \( t \). We consider forecasting horizons of 8, 12, 16 and 20 quarters. Forecasting inflation at longer horizons is crucial for policymakers since it is known that monetary policy action has an effect on inflation with several lags. The model versions are similar to the ones in Table 7. Models 1 and 3 incorporate the yield spread while models 2 and 4 include the decomposed spread. Model 2 does not include control variables and model 4 includes both past inflation (\( \pi_t^{(-4)} \)) and the short-term interest rate (\( y_t^{(1)} \)) as control variables.

The results for model 1 in Table 9 indicate that although yield spreads significantly predict inflation changes the resulting adjusted \( R^2 \) is relatively low. We now compare the impact on the adjusted \( R^2 \) of the yield spread decomposition (model 2) and of the inclusion of control variables (model 3). The use of the decomposed spread leads to a higher increase in the \( R^2 \)’s for horizons of 2 and 3 years but the inclusion of control variables has a higher effect for longer horizons (4 and 5 years). Even with the inclusion of control variables, the use of a decomposed yield spread (model 4) leads to an increase in the adjusted \( R^2 \) between 5 and 20 percentage
points for different forecasting horizons, reaching 44% for a 12-quarter horizon.

Insert Table 9

One reason for the difference in the relevance of the spread decomposition in the regressions for real economic activity and inflation is the fact that for the latter the expectations and the term premium components have consistently opposite signs and are in most cases statistically significant. While the expectations component has a positive association with future inflation, the term premium component correlates negatively with inflation; a positive expectations component of the spread signals future increases in inflation (in line with the Fisher parity), while a positive term premium indicates a decrease in future inflation. These differences in sign can obviously not be captured by the spread itself.

The results for model 3 also show that the control variables drive out the predictive power of the yield spread. But model 4 shows that this is not the case once the yield spread is decomposed. So, while yield spreads *per se* contain valuable information, decomposing the spread into expectations and term premium components clearly improves the forecasting potential of the predictive regressions for inflation changes. Our results show that any interpretation of the yield spread in terms of long-run inflation expectation can be seriously biased by the time-varying risk premia.

Finally, we analyze the time evolution of the predictive content of the yield spread and its components for inflation changes (Figure 6). Each plot in this figure shows the adjusted $R^2$
over time for a certain predictive horizon and the corresponding yield spread. As in Figure 5, the plots indicate the resulting adjusted $R^2$ for a sample ending on the shown date with the EFM model being reestimated at every quarter using an expanding window. The results for the inflation changes show a slight decrease over time in the predictive power of the yield spread and its components to forecast inflation changes. We also note a striking improvement in the adjusted $R^2$ simply by decomposing the spread in its two components. This is especially the case for a 8-quarter horizon. As mentioned before, this is due to the fact that the expectations and the term premium components obtain opposite signs in the regressions. For a 20-quarter horizon, once you allow for control variables the gain from decomposing the spread is marginal.

5 Conclusion

In this paper, we use the EMF model proposed by Dewachter and Iania (2010) to study the risk and term premia in the U.S. bond market. This model extends standard MF models by including next to the standard macroeconomic factors a set of financial factors. The latter include liquidity and risk premium factors. By including these factors the model is able to capture in a better way the additional non-macroeconomic drivers of the yield curve.

The estimation results indicate that risk premia in the U.S. market display significant time variation and strong collinearity across the maturity spectrum. The former is a clear indication
that the expectation hypothesis fails. More importantly, a variance decomposition implied by the EMF model singles out financial factors as the main drivers behind bond premia. In particular, we find that risk premium shocks dominate. This finding is in line with the recent literature indicating that macroeconomic factors cannot account for the time variation in risk premia. The significant collinearity of bond premia suggests that a few factors drive the entire term structure of risk premia. We find that one factor, closely related to the CP factor \cite{CochranePiazzesi2005}, is responsible for most of the variation in bond premia.

We use the EMF model to decompose the yield spread into expectations and term premium components. This decomposition is used to forecast economic activity and inflation. Although the decomposition does not seem important to forecast economic activity it is crucial to forecast inflation for most forecasting horizons. Also, in general, the inclusion of control variables such as the short-term interest rate and lagged variables does not drive out the predictive power of the yield spread decomposition.
References


Swanson, E. (2007). What we do and don’t know about the term premium. FRBSF Economic Letter 21, 1–3.

### Table 1: Prior and Posterior Distribution - Prices of Risk

<table>
<thead>
<tr>
<th></th>
<th>Prior</th>
<th>Posterior</th>
<th>5 %</th>
<th>16 %</th>
<th>50 %</th>
<th>84 %</th>
<th>95 %</th>
<th>Mean</th>
<th>Mode</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Distr</td>
<td>Mean</td>
<td>Stdev</td>
<td>Mean</td>
<td>Stdev</td>
<td>Mean</td>
<td>Stdev</td>
<td>Mean</td>
<td>Stdev</td>
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<tr>
<td><strong>A_0 (1)</strong></td>
<td>~N</td>
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<td>1.000</td>
<td>0.130</td>
<td>0.427</td>
<td>0.888</td>
<td>1.380</td>
<td>1.734</td>
<td>0.865</td>
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<td>-1.259</td>
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<td><strong>A_0 (5)</strong></td>
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<td>-1.097</td>
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<td>1.000</td>
<td>0.398</td>
<td>0.476</td>
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<td><strong>A_0 (8)</strong></td>
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<td>1.000</td>
<td>-2.448</td>
<td>-1.903</td>
<td>-1.061</td>
<td>-0.230</td>
<td>0.390</td>
<td>-1.034</td>
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<tr>
<td><strong>A_1 (1, 6)</strong></td>
<td>~N</td>
<td>0.000</td>
<td>25.000</td>
<td>-5.347</td>
<td>7.851</td>
<td>26.515</td>
<td>44.379</td>
<td>55.710</td>
<td>26.001</td>
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<td><strong>A_1 (2, 6)</strong></td>
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<td>25.000</td>
<td>-35.621</td>
<td>-18.808</td>
<td>4.262</td>
<td>27.527</td>
<td>42.581</td>
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<td><strong>A_1 (3, 6)</strong></td>
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<td>25.000</td>
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<td>-57.537</td>
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<td>-47.722</td>
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<td><strong>A_1 (4, 6)</strong></td>
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<td>25.000</td>
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<td>-8.077</td>
<td>9.444</td>
<td>26.279</td>
<td>38.876</td>
<td>8.397</td>
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<tr>
<td><strong>A_1 (5, 6)</strong></td>
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<td>0.000</td>
<td>25.000</td>
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<td>-0.159</td>
<td>15.947</td>
<td>29.594</td>
<td>40.290</td>
<td>12.501</td>
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<tr>
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<td>25.000</td>
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<td>-6.939</td>
<td>-0.839</td>
<td>5.380</td>
<td>10.295</td>
<td>-1.052</td>
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</table>

**Note:** This table reports the prior and posterior density of $A_0$ and $A_1$ in equation (6). The second to fourth columns report the type of distribution, mean and standard deviation of the prior, respectively. The fifth to ninth columns report the 5-th, 16-th, 50-th, 84-th, and 95-th percentile of the posterior distribution, respectively. The last two columns report the mean and mode of the posterior distribution. All results are obtained using the Metropolis-Hastings algorithm.
Table 2: Excess returns: in-sample and out-of-sample analysis

### In-sample statistics

<table>
<thead>
<tr>
<th>maturity (n)</th>
<th>Adj. $R^2$: 4-qtr holding period ($k$)</th>
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<tbody>
<tr>
<td></td>
<td>8 qtr</td>
</tr>
<tr>
<td>CP</td>
<td>29.90%</td>
</tr>
<tr>
<td>EMF</td>
<td>36.15%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>maturity (n)</th>
<th>Adj. $R^2$: 8-qtr holding period ($k$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8 qtr</td>
</tr>
<tr>
<td>CP</td>
<td>-</td>
</tr>
<tr>
<td>EMF</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>maturity (n)</th>
<th>Adj. $R^2$: 16-qtr holding period ($k$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8 qtr</td>
</tr>
<tr>
<td>CP</td>
<td>-</td>
</tr>
<tr>
<td>EMF</td>
<td>-</td>
</tr>
</tbody>
</table>

### Out-of-sample statistics

<table>
<thead>
<tr>
<th>maturity (n)</th>
<th>4-qtr holding period ($k$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8 qtr</td>
</tr>
<tr>
<td>EMF (RMSE)</td>
<td>1.54%</td>
</tr>
<tr>
<td>CP (RMSE)/EMF (RMSE)</td>
<td>1.03</td>
</tr>
<tr>
<td>RW (RMSE)/EMF (RMSE)</td>
<td>1.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>maturity (n)</th>
<th>8-qtr holding period ($k$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8 qtr</td>
</tr>
<tr>
<td>EMF (RMSE)</td>
<td>-</td>
</tr>
<tr>
<td>CP (RMSE)/EMF (RMSE)</td>
<td>-</td>
</tr>
<tr>
<td>RW (RMSE)/EMF (RMSE)</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: This table reports in-sample (top three panels) and out-of-sample (bottom two panels) statistics for the excess bond returns. The top three panels report the adjusted $R^2$ of the following regression:

$$r_{x,t}^{(n)} = \alpha + \beta f_t^{(n)} + \varepsilon_{t+k}, \quad n = 8, 12, 16, 20 \text{ qtr}; \quad k = 4, 8, 16 \text{ qtr}; \quad n > k$$

where $r_{x,t}^{(n)}$ denotes the realized excess return (in excess of the $k$-quarter risk-free rate) of buying an $n$-quarter bond at time $t$ and selling it back after $k$ quarters, and $f_t^{(n)}$ represents the unsmoothed Cochrane and Piazzesi (2005) factor (CP) or the EMF model-implied expected excess returns (first and second rows in each panel). When regressing the realized excess returns on the model-implied ones we fixed the coefficients $\alpha$ and $\beta$ to 0 and 1, respectively. The sample period goes from 1960:Q1 to 2008:Q4.

The bottom two panels report the out-of-sample forecasts of the realized excess returns using the EMF model, the CP model, and the random walk model (RW). For the EMF and the CP models, the forecasts are obtained (i) by estimating the models over the period 1960:Q1-1995:Q4 and (ii) by producing the model-implied forecasts of the excess returns for the period 1996:Q1-2008:Q4. Every quarter the information is updated and the models are reestimated. The first row of the panel reports the root mean squared error (RMSE) of the EMF model. The following two rows present the ratios of the RMSE of the CP model over the EMF model and of the RW model over the EMF model, respectively.
Table 3: Unbiasedness of expected excess returns

<table>
<thead>
<tr>
<th>4-qtr holding period (k)</th>
<th>maturity (n)</th>
<th>8 qtr</th>
<th>12 qtr</th>
<th>16 qtr</th>
<th>20 qtr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>α</td>
<td>0.000</td>
<td>0.001</td>
<td>0.002</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.006)</td>
</tr>
<tr>
<td></td>
<td>β</td>
<td>1.058</td>
<td>1.031</td>
<td>1.009</td>
<td>0.969</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.155)</td>
<td>(0.177)</td>
<td>(0.185)</td>
<td>(0.172)</td>
</tr>
<tr>
<td>p-value (α = 0, β = 1)</td>
<td></td>
<td>0.915</td>
<td>0.924</td>
<td>0.950</td>
<td>0.984</td>
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<table>
<thead>
<tr>
<th>8-qtr holding period (k)</th>
<th>maturity (n)</th>
<th>8 qtr</th>
<th>12 qtr</th>
<th>16 qtr</th>
<th>20 qtr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>α</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td></td>
<td>β</td>
<td>1.108</td>
<td>1.061</td>
<td>1.003</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
<td>(0.169)</td>
<td>(0.195)</td>
<td>(0.284)</td>
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<tr>
<td>p-value (α = 0, β = 1)</td>
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<td>-</td>
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<td>0.389</td>
<td>0.394</td>
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<table>
<thead>
<tr>
<th>16-qtr holding period (k)</th>
<th>maturity (n)</th>
<th>8 qtr</th>
<th>12 qtr</th>
<th>16 qtr</th>
<th>20 qtr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>α</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>β</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.208</td>
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<td>-</td>
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<td>(0.160)</td>
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<tr>
<td>p-value (α = 0, β = 1)</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>0.386</td>
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**Note:** This table reports coefficients and respective Newey-West standard errors of the following regression:

\[ r_{x, t+k}^{(n)} = \alpha + \beta E_t \left[ r_{x, t+k}^{(n)} \right] + \varepsilon_{t+k}, \quad n = 8, 12, 16, 20 \text{ qtr}; \quad k = 4, 8, 16 \text{ qtr}; \quad n > k \]

where \( r_{x, t+k}^{(n)} \) denotes the realized excess return (in excess of the \( k \)-quarter risk-free rate) of buying a \( n \)-quarter bond at time \( t \) and selling it back after \( k \) quarters, and \( E_t \left[ r_{x, t+k}^{(n)} \right] \) represents the EMF model-implied expected excess return as given by equation (15). In the last row of each panel, we report the p-value of the joint test \( \alpha = 0 \) and \( \beta = 1 \). The standard errors of the coefficients are in parentheses. The sample period goes from 1960:Q1 to 2008:Q4.
Table 4: Variance decomposition

Panel (a): Bond risk premia

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<tr>
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<tbody>
<tr>
<td>1 qtr</td>
<td>0.6%</td>
<td>0.2%</td>
<td>0.0%</td>
<td>19.2%</td>
<td>79.8%</td>
<td>0.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>4 qtr</td>
<td>2.6%</td>
<td>1.1%</td>
<td>5.0%</td>
<td>12.3%</td>
<td>78.8%</td>
<td>0.2%</td>
<td>0.0%</td>
</tr>
<tr>
<td>40 qtr</td>
<td>2.5%</td>
<td>1.4%</td>
<td>17.2%</td>
<td>19.1%</td>
<td>59.6%</td>
<td>0.3%</td>
<td>0.0%</td>
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<tr>
<td>100 qtr</td>
<td>2.5%</td>
<td>1.4%</td>
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<td>59.6%</td>
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20-qtr bond (4-qtr holding period)

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</thead>
<tbody>
<tr>
<td>1 qtr</td>
<td>0.5%</td>
<td>0.2%</td>
<td>0.0%</td>
<td>20.2%</td>
<td>79.1%</td>
<td>0.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>4 qtr</td>
<td>2.4%</td>
<td>1.0%</td>
<td>4.5%</td>
<td>13.1%</td>
<td>78.8%</td>
<td>0.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>40 qtr</td>
<td>2.4%</td>
<td>1.3%</td>
<td>16.5%</td>
<td>19.3%</td>
<td>60.3%</td>
<td>0.3%</td>
<td>0.0%</td>
</tr>
<tr>
<td>100 qtr</td>
<td>2.4%</td>
<td>1.3%</td>
<td>16.5%</td>
<td>19.3%</td>
<td>60.3%</td>
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</table>

Panel (b): Expectations component

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</tr>
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<tbody>
<tr>
<td>1 qtr</td>
<td>1.8%</td>
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<td>0.2%</td>
<td>0.1%</td>
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<tr>
<td>4 qtr</td>
<td>3.5%</td>
<td>2.7%</td>
<td>79.8%</td>
<td>12.8%</td>
<td>0.0%</td>
<td>1.0%</td>
<td>0.2%</td>
</tr>
<tr>
<td>40 qtr</td>
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<td>30.6%</td>
<td>3.1%</td>
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<tr>
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<td>2.1%</td>
<td>26.4%</td>
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<td>55.3%</td>
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</table>

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<td>83.8%</td>
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</table>

Panel (c): Term premium component

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<td>2.5%</td>
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<td>0.1%</td>
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<td>12.6%</td>
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<td>4 qtr</td>
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<td>0.9%</td>
<td>20.7%</td>
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<td>65.2%</td>
<td>0.6%</td>
<td>0.0%</td>
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<td>23.4%</td>
<td>29.3%</td>
<td>41.4%</td>
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<td>0.0%</td>
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</tbody>
</table>

Note: This table reports the forecasting error variance decomposition (computed at the mode of the posterior distribution of the parameters) of the 4-quarter bond premia of 8- and 20-quarter maturity bonds (Panel A), of the average expected 1-quarter interest rate over 4 quarters and 40 quarters (Panel B), and of the 4-quarter and 40-quarter term premium (Panel C). Sup. sh.: supply shocks; Dem. sh.: demand shocks; Pol. rate sh.: policy rates shocks; Liq. sh.: flight-to-quality and credit-crunch shocks; LR inf. sh.: long-run inflation shocks; and Eq. real rate sh.: equilibrium real rate shocks.
Table 5: Forecasting GDP Growth

<table>
<thead>
<tr>
<th>Model 1</th>
<th>( g_{t+1} = \alpha + \beta_{t} + \varepsilon_{t+1} )</th>
<th>( g_{t+1} = \beta_{t} + \varepsilon_{t+1} )</th>
<th>( g_{t+1} = \beta_{EC} + \varepsilon_{t+1} )</th>
<th>( g_{t+1} = \beta_{TP} + \varepsilon_{t+1} )</th>
<th>( g_{t+1} = \gamma_{t} + \delta_{t} + \varepsilon_{t+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.026 (0.004)</td>
<td>0.023 (0.004)</td>
<td>0.025 (0.004)</td>
<td>0.026 (0.004)</td>
<td>0.026 (0.004)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>1.362 (0.224)</td>
<td>0.593 (0.199)</td>
<td>1.462 (0.214)</td>
<td>1.090 (0.241)</td>
<td>0.345 (0.148)</td>
</tr>
<tr>
<td>Adj.-( R^2 )</td>
<td>0.044 (0.076)</td>
<td>0.098 (0.076)</td>
<td>0.078 (0.110)</td>
<td>0.072 (0.160)</td>
<td>0.027 (0.025)</td>
</tr>
</tbody>
</table>

Note: This table reports the predictive coefficients, standard errors, and adjusted \( R^2 \) of four models to forecast GDP growth (\( g_{t+1} \)) over a horizon of 1, 4, and 8 quarters. Model 1 relates the GDP growth to the GDP growth and risk-free rate (\( \gamma_{t} \)). Model 2 links the GDP growth to the EMF model-implied 4-, 20-, and 40-quarter expectations (\( \hat{y}_{t}^{(4)} \), \( \hat{y}_{t}^{(20)} \), \( \hat{y}_{t}^{(40)} \)) and term premium (\( \chi_{t} \)) components of the yield spread. Model 3 relates the GDP growth to the 4-, 20-, and 40-quarter yield spreads, the GDP growth between \( t \) and \( t+1 \), and the 1-quarter risk-free rate (\( \gamma_{t} \)). Model 4 forecasts the GDP growth by means of the EMF model-implied 4-, 20-, and 40-quarter expectations and term premium components of the yield spread, the GDP growth between \( t-1 \) and \( t \) quarter, and the 1-quarter risk-free rate (\( \gamma_{t} \)). The standard errors of the coefficients are in parentheses. We estimate the model over the period 1960:Q1-2008:Q4.
Table 6: Forecasting output gap

<table>
<thead>
<tr>
<th>Model</th>
<th>horizon ((k))</th>
<th>maturity ((n))</th>
<th>(y_{t+k} = \alpha + \beta \text{Spr}<em>{t}^{(n)} + \varepsilon</em>{t+k})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 qtr</td>
<td>4 qtr</td>
<td>4 qtr</td>
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<tr>
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<td>20 qtr</td>
<td>20 qtr</td>
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<tr>
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<tr>
<td></td>
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<td>20 qtr</td>
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<td>20 qtr</td>
<td>20 qtr</td>
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<td>40 qtr</td>
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<tr>
<td></td>
<td>4 qtr</td>
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</table>

<table>
<thead>
<tr>
<th>Model 1</th>
<th>horizon ((k))</th>
<th>maturity ((n))</th>
<th>(y_{t+k} = \alpha + \beta \text{Spr}<em>{t}^{(n)} + \varepsilon</em>{t+k})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>-0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>-0.309</td>
<td>-0.543</td>
<td>-0.394</td>
</tr>
<tr>
<td></td>
<td>(0.546)</td>
<td>(0.271)</td>
<td>(0.197)</td>
</tr>
<tr>
<td>Adj.-(R^2)</td>
<td>0.001</td>
<td>0.063</td>
<td>0.052</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model 2</th>
<th>horizon ((k))</th>
<th>maturity ((n))</th>
<th>(y_{t+k} = \alpha + \beta^{EC} \text{Spr}<em>{t}^{(n)} + \gamma y</em>{t} + \delta y_{t}^{(1)} + \varepsilon_{t+k})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>-0.002</td>
<td>0.004</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>(\beta^{EC})</td>
<td>2.215</td>
<td>0.064</td>
<td>-0.035</td>
</tr>
<tr>
<td></td>
<td>(0.555)</td>
<td>(0.198)</td>
<td>(0.173)</td>
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<tr>
<td>(\beta^{TP})</td>
<td>-2.589</td>
<td>-0.852</td>
<td>-0.799</td>
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<tr>
<td></td>
<td>(0.633)</td>
<td>(0.253)</td>
<td>(0.256)</td>
</tr>
<tr>
<td>Adj.-(R^2)</td>
<td>0.251</td>
<td>0.252</td>
<td>0.242</td>
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<table>
<thead>
<tr>
<th>Model 3</th>
<th>horizon ((k))</th>
<th>maturity ((n))</th>
<th>(y_{t+k} = \alpha + \beta \text{Spr}<em>{t}^{(n)} + \gamma y</em>{t} + \delta y_{t}^{(1)} + \varepsilon_{t+k})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>0.002</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.227</td>
<td>0.180</td>
<td>0.133</td>
</tr>
<tr>
<td></td>
<td>(0.123)</td>
<td>(0.056)</td>
<td>(0.052)</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.934</td>
<td>0.967</td>
<td>0.964</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.032)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>-0.064</td>
<td>-0.036</td>
<td>-0.031</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.032)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>Adj.-(R^2)</td>
<td>0.871</td>
<td>0.874</td>
<td>0.872</td>
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<table>
<thead>
<tr>
<th>Model 4</th>
<th>horizon ((k))</th>
<th>maturity ((n))</th>
<th>(y_{t+k} = \alpha + \beta^{EC} \text{Spr}<em>{t}^{(n)} + \gamma y</em>{t} + \delta y_{t}^{(1)} + \varepsilon_{t+k})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>-0.000</td>
<td>-0.005</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>(\beta^{EC})</td>
<td>0.658</td>
<td>0.374</td>
<td>0.301</td>
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<tr>
<td></td>
<td>(0.267)</td>
<td>(0.084)</td>
<td>(0.075)</td>
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<tr>
<td>(\beta^{TP})</td>
<td>-0.342</td>
<td>-0.229</td>
<td>-0.190</td>
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<tr>
<td></td>
<td>(0.288)</td>
<td>(0.081)</td>
<td>(0.074)</td>
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<tr>
<td>(\gamma)</td>
<td>0.918</td>
<td>0.943</td>
<td>0.955</td>
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<td>(0.037)</td>
<td>(0.033)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>-0.031</td>
<td>0.045</td>
<td>0.052</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.041)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>Adj.-(R^2)</td>
<td>0.877</td>
<td>0.880</td>
<td>0.878</td>
</tr>
</tbody>
</table>

Note: This table reports the predictive coefficients, standard errors, and adjusted \(R^2\) of four models forecasting the output gap over a horizon of 1, 4, and 8 quarters. Model 1 relates future output gap to the current 4, 20 and 40-quarter term spread \((\text{Spr}_{t}^{(n)} = y_{t}^{(b)} - y_{t}^{(q)})\). Model 2 links future output gap to the EMF model-implied 4, 20 and 40-quarter expectations \((\text{Spr}_{t}^{(n)} + \chi_{t}^{(n)})\) and term premium \((\chi_{t}^{(n)})\) components of the yield spread at time \(t\). Model 3 relates future output gap to the 4, 20 and 40-quarter yield spreads, output gap at time \(t\) \((y_t)\), and the 1-quarter risk-free rate \((y^{(1)}_t)\). Model 4 links future output gap to the EMF model-implied 4, 20 and 40-quarter expectations and term premium components of the yield spread, current output gap, and the 1-quarter risk-free rate. The standard errors of the coefficients are in parentheses. We estimate the model over the period 1960:Q1-2008:Q4.
### Table 7: Forecasting Inflation

<table>
<thead>
<tr>
<th>Model 1</th>
<th>( \pi_{t+k} - \pi_{t-4,q-1} = \alpha + \beta \text{SP}<em>{1}^{(k)} + \epsilon</em>{t+k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizon (k)</td>
<td>maturity (n)</td>
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<tr>
<td>( \alpha )</td>
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<tr>
<td>( \beta )</td>
<td>0.042</td>
</tr>
<tr>
<td>Adj.-( R^2 )</td>
<td>-0.005</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model 2</th>
<th>( \pi_{t+k} - \pi_{t-3,q} = \alpha + \beta^{EC} \left( \text{SP}<em>{1}^{(k)} + \chi</em>{1}^{(k)} \right) + \beta^{TP} \chi_{1}^{(k)} + \epsilon_{t+k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizon (k)</td>
<td>maturity (n)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.001</td>
</tr>
<tr>
<td>( \beta^{EC} )</td>
<td>0.535</td>
</tr>
<tr>
<td>( \beta^{TP} )</td>
<td>-0.777</td>
</tr>
<tr>
<td>Adj.-( R^2 )</td>
<td>-0.002</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model 3</th>
<th>( \pi_{t+k} - \pi_{t-3,q} = \alpha + \beta \text{SP}<em>{1}^{(k)} + \gamma (\pi</em>{t} - \pi_{t-4,q-1}) + \delta y_{1}^{(k)} + \epsilon_{t+k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizon (k)</td>
<td>maturity (n)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.003</td>
</tr>
<tr>
<td>( \beta^{EC} )</td>
<td>0.043</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.350</td>
</tr>
<tr>
<td>( \delta )</td>
<td>-0.053</td>
</tr>
<tr>
<td>Adj.-( R^2 )</td>
<td>0.136</td>
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<table>
<thead>
<tr>
<th>Model 4</th>
<th>( \pi_{t+k} - \pi_{t-3,q} = \alpha + \beta^{EC} \left( \text{SP}<em>{1}^{(k)} + \chi</em>{1}^{(k)} \right) + \beta^{TP} \chi_{1}^{(k)} + \gamma (\pi_{t} - \pi_{t-4,q-1}) + \delta y_{1}^{(k)} + \epsilon_{t+k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizon (k)</td>
<td>maturity (n)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.001</td>
</tr>
<tr>
<td>( \beta^{EC} )</td>
<td>0.340</td>
</tr>
<tr>
<td>( \beta^{TP} )</td>
<td>-0.536</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.285</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.008</td>
</tr>
<tr>
<td>Adj.-( R^2 )</td>
<td>0.152</td>
</tr>
</tbody>
</table>

**Note:** This table reports the predictive coefficients, standard errors, and adjusted \( R^2 \) of four models forecasting the deviation from inflation \( k \) periods ahead and the average of quarterly inflation over the past four quarters, \( \pi_{t-3,q} \), over a horizon of 1, 4 and 8 quarters. Model 1 relates future inflation deviations to the current 4, 20 and 40-quarter term spread \( \left( \text{SP}_{1}^{(k)} = y_{1}^{(k)} - y_{1}^{(k)} \right) \). Model 2 links future inflation deviations to the EMF model-implied 4, 20 and 40-quarter expectations \( \left( \text{SP}_{1}^{(k)} \right) \) and term premium \( \left( \chi_{1}^{(k)} \right) \) components of the yield spread at time \( t \). Model 3 relates future inflation deviations to the 4, 20 and 40-quarter yields spreads, average inflation over the past four periods \( \left( \pi_{t-4,q-1} \right) \), and the 1-quarter risk-free rate \( \left( y_{1}^{(k)} \right) \). Model 4 links future inflation deviations to the EMF model-implied 4, 20 and 40-quarter expectations and term premium components of the yield spread, average inflation over the past four periods \( \left( \pi_{t-4,q-1} \right) \), and the 1-quarter risk-free rate. The standard errors of the coefficients are in parentheses. We estimate the model over the period 1960:Q1-2008:Q4.
Table 8: Forecasting inflation: the impact of Greenbook forecasts

<table>
<thead>
<tr>
<th>Model 1</th>
<th>1 qtr</th>
<th>4 qtr</th>
<th>20 qtr</th>
<th>40 qtr</th>
<th>4 qtr</th>
<th>20 qtr</th>
<th>40 qtr</th>
<th>4 qtr</th>
<th>20 qtr</th>
<th>40 qtr</th>
<th>4 qtr</th>
<th>20 qtr</th>
<th>40 qtr</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizon (k)</td>
<td>4 qtr</td>
<td>20 qtr</td>
<td>40 qtr</td>
<td>4 qtr</td>
<td>20 qtr</td>
<td>40 qtr</td>
<td>4 qtr</td>
<td>20 qtr</td>
<td>40 qtr</td>
<td>4 qtr</td>
<td>20 qtr</td>
<td>40 qtr</td>
<td>4 qtr</td>
</tr>
<tr>
<td>α</td>
<td>0.003</td>
<td>0.005</td>
<td>0.006</td>
<td>0.007</td>
<td>0.009</td>
<td>0.010</td>
<td>0.011</td>
<td>0.012</td>
<td>0.012</td>
<td>0.013</td>
<td>0.014</td>
<td>0.015</td>
<td>0.016</td>
</tr>
<tr>
<td>θ</td>
<td>-0.035</td>
<td>-0.031</td>
<td>-0.028</td>
<td>-0.025</td>
<td>-0.022</td>
<td>-0.019</td>
<td>-0.017</td>
<td>-0.015</td>
<td>-0.015</td>
<td>-0.015</td>
<td>-0.015</td>
<td>-0.015</td>
<td>-0.015</td>
</tr>
<tr>
<td>γ</td>
<td>0.187</td>
<td>0.167</td>
<td>0.147</td>
<td>0.127</td>
<td>0.107</td>
<td>0.087</td>
<td>0.067</td>
<td>0.047</td>
<td>0.027</td>
<td>0.007</td>
<td>-0.007</td>
<td>-0.007</td>
<td>-0.007</td>
</tr>
<tr>
<td>δ</td>
<td>0.133</td>
<td>0.121</td>
<td>0.111</td>
<td>0.101</td>
<td>0.091</td>
<td>0.081</td>
<td>0.071</td>
<td>0.061</td>
<td>0.051</td>
<td>0.041</td>
<td>0.031</td>
<td>0.021</td>
<td>0.011</td>
</tr>
<tr>
<td>Adj R²</td>
<td>0.162</td>
<td>0.160</td>
<td>0.158</td>
<td>0.156</td>
<td>0.154</td>
<td>0.152</td>
<td>0.150</td>
<td>0.148</td>
<td>0.146</td>
<td>0.144</td>
<td>0.142</td>
<td>0.140</td>
<td>0.138</td>
</tr>
</tbody>
</table>

| Model 2 | 1 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr |
|---------|-------|-------|--------|--------|-------|--------|--------|-------|--------|--------|-------|--------|--------|-------|--------|--------|-------|--------|--------|-------|--------|--------|-------|--------|
| horizon (k) | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr | 4 qtr | 20 qtr | 40 qtr |
| α | 0.002 | 0.002 | 0.002 | 0.002 | 0.003 | 0.002 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| θ | -0.136 | -0.152 | -0.171 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 | -0.174 |
| γ | 0.382 | 0.400 | 0.418 | 0.435 | 0.454 | 0.472 | 0.490 | 0.509 | 0.527 | 0.546 | 0.564 | 0.583 | 0.602 | 0.621 | 0.640 | 0.659 | 0.678 | 0.697 | 0.716 | 0.735 | 0.754 | 0.773 |
| δ | -0.339 | -0.354 | -0.370 | -0.385 | -0.400 | -0.415 | -0.430 | -0.445 | -0.460 | -0.475 | -0.490 | -0.505 | -0.520 | -0.535 | -0.550 | -0.565 | -0.580 | -0.595 | -0.610 | -0.625 | -0.640 | -0.655|
| Adj R² | 0.131 | 0.126 | 0.121 | 0.117 | 0.112 | 0.107 | 0.102 | 0.097 | 0.092 | 0.087 | 0.082 | 0.077 | 0.072 | 0.067 | 0.062 | 0.057 | 0.052 | 0.047 | 0.042 | 0.037 | 0.032 | 0.027 |

Note: This table reports the predictive coefficients, standard errors, and adjusted R² of two models forecasting the deviation in inflation k periods ahead and the average of quarterly inflation over the past four quarters, \(\pi_{t+k} - \pi_t = \alpha + \beta \pi_{t+k} + \gamma \pi_t + \delta \pi_{t+k} + \epsilon_{t+k}\), over a horizon of 1 and 4 quarters. Model 1 relates future inflation deviations to the inflation forecast (\(\pi_{t+k}\)), a first-inflation deviation (\(\pi_t\)), and the 1-quarter riskfree interest rate (\(\pi_t\)). Model 2 replaces the yield spread for its expectations (\(\delta_{t+k}\)) and term premium (\(\pi_t\)).

The standard errors of the coefficients are in parentheses. We estimate the model over the period 1974:Q2-2005:Q4.
Table 9: Forecasting forward inflation changes

<table>
<thead>
<tr>
<th>Model</th>
<th>( \pi_{t+k}^{(k)} - \pi_{t+k}^{(4)} = \alpha + \beta \pi_t^{(k)} + \varepsilon_{t+k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>8 qtr</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.003</td>
</tr>
<tr>
<td>( (0.003) )</td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.419</td>
</tr>
<tr>
<td>( (0.203) )</td>
<td>(0.208)</td>
</tr>
<tr>
<td>Adj.-( R^2 )</td>
<td>0.063</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model 2</th>
<th>( \pi_{t+k}^{(k)} - \pi_{t+k}^{(4)} = \alpha + \beta^{EC} \pi_t^{(k)} + \delta \pi_t^{(4)} + \varepsilon_{t+k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>8 qtr</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.001</td>
</tr>
<tr>
<td>( (0.001) )</td>
<td>(0.001)</td>
</tr>
<tr>
<td>( \beta^{EC} )</td>
<td>1.045</td>
</tr>
<tr>
<td>( (0.239) )</td>
<td>(0.228)</td>
</tr>
<tr>
<td>( \beta^{TP} )</td>
<td>-0.718</td>
</tr>
<tr>
<td>( (0.157) )</td>
<td>(0.144)</td>
</tr>
<tr>
<td>Adj.-( R^2 )</td>
<td>0.302</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model 3</th>
<th>( \pi_{t+k}^{(k)} - \pi_{t+k}^{(4)} = \alpha + \beta \pi_t^{(k)} + \gamma \pi_t^{(-4)} + \delta \pi_t^{(4)} + \varepsilon_{t+k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>8 qtr</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.004</td>
</tr>
<tr>
<td>( (0.002) )</td>
<td>(0.003)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.253</td>
</tr>
<tr>
<td>( (0.167) )</td>
<td>(0.183)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>-0.012</td>
</tr>
<tr>
<td>( (0.056) )</td>
<td>(0.076)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>-0.075</td>
</tr>
<tr>
<td>( (0.027) )</td>
<td>(0.039)</td>
</tr>
<tr>
<td>Adj.-( R^2 )</td>
<td>0.193</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model 4</th>
<th>( \pi_{t+k}^{(k)} - \pi_{t+k}^{(4)} = \alpha + \beta^{EC} \pi_t^{(k)} + \delta \pi_t^{(4)} + \varepsilon_{t+k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>8 qtr</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-0.005</td>
</tr>
<tr>
<td>( (0.002) )</td>
<td>(0.003)</td>
</tr>
<tr>
<td>( \beta^{EC} )</td>
<td>1.707</td>
</tr>
<tr>
<td>( (0.371) )</td>
<td>(0.289)</td>
</tr>
<tr>
<td>( \beta^{TP} )</td>
<td>-1.713</td>
</tr>
<tr>
<td>( (0.384) )</td>
<td>(0.311)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>-0.137</td>
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<tr>
<td>( (0.051) )</td>
<td>(0.062)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.172</td>
</tr>
<tr>
<td>( (0.057) )</td>
<td>(0.077)</td>
</tr>
<tr>
<td>Adj.-( R^2 )</td>
<td>0.388</td>
</tr>
</tbody>
</table>

**Note:** This table reports the predictive coefficients, standard errors, and adjusted \( R^2 \) of four models forecasting the change in inflation (growth of the GDP deflator) over a horizon of 8, 12, 16 and 20 quarters. The methodology is directly related to Estrella and Mishkin (1997) and Mishkin (1990). The regressand is the difference between the future \( k \)-quarter inflation rate from time \( t \) to \( t+k \) (\( \pi_t^{(k)} \)) and the future 4-quarter inflation rate from \( t \) to \( t+4 \) (\( \pi_t^{(4)} \)). Model 1 relates the change in inflation to the current 8, 12, 16 and 20-quarter yield spread over the 4-quarter yield (\( S_{t+k}^{(k)} = \pi_t^{(k)} - \pi_t^{(4)} \)). Model 2 links the change in inflation to the EMF model-implied 8, 12, 16 and 20-quarter expectations (\( S_{t+k}^{(k)} \)) and term premium (\( \pi_t^{(4)} \)) components of the yield spread at time \( t \). Model 3 relates the change in inflation to the 8, 12, 16 and 20-quarter yield spreads, past inflation between \( t-4 \) quarters and \( t-1 \) (\( \pi_t^{(-4)} \)), and the 1-quarter risk free rate (\( y_t^{(1)} \)). Model 4 forecasts the change in inflation by means of the EMF model-implied 8, 12, 16 and 20-quarter expectations and term premium components of the yield spread, past 4 quarters inflation, and the 1-quarter risk free rate. The standard errors of the coefficients are in parentheses. We estimate the model over the period 1960:Q1-2008:Q4.
Figure 1: Excess return: Expected vs. realized

Note: This figure compares the EMF model-implied expected excess return (bond premium, continuous line) with the realized excess return (dashed line). The holding period is 4 quarters for bonds with maturities of 8, 12, 16 and 20 quarters.
Figure 2: RETURN-FORECASTING FACTOR: CP VS. EMF FACTOR

Note: This figure compares the Cochrane and Piazzesi (2005) factor (CP) with the EMF risk premium factor. Since the original CP factor is computed using monthly data and we work with quarterly frequencies, we compute the CP factor on a monthly basis and for each quarter we take the average of the monthly series. The correlation between our factor and the CP factor is 0.67.
Figure 3: Ten-year yield: fitted value, expectations component and term premium component

Note: The top panel of this figure plots the 40-quarter fitted yield. The middle panel depicts the EMF model-implied expected average 1-quarter yield over a period of 40 quarters. The bottom panel compares the EMF model-implied term premium for the 40-quarter bond (continuous line) with the term premium of Kim and Wright (2005) (dashed line).
Figure 4: **Ten-year spread: fitted value, expectations component and term premium component**

**Note:** The top panel of this figure plots the fitted spread of the 40-quarter yield less the 1-quarter yield. The middle panel depicts the EMF model-implied expected average 1-quarter yield over a period of 40 quarters minus the 1-quarter yield. The bottom panel compares the EMF model-implied term premium for the 40-quarter bond (continuous line) with the term premium of Kim and Wright (2005) (dashed line).
Figure 5: Forecasting GDP growth, Expanding window (R-squared)

Note: Each plot of this figure shows the adjusted $R^2$ over time for a certain predictive horizon using a certain yield spread. The rows of panels define the predictive horizon (1, 4, and 8 quarters) and the columns of panels the maturity of the yield spread used in the regression (4, 20, and 40 quarters). The date on the horizontal axis determines the end date of the sample period. The first point in each graph indicates the adjusted $R^2$ for the period 1960:Q1-1995:Q4. The EFM model is reestimated at every quarter using an expanding window.
Figure 6: **Forecasting Inflation Changes, Expanding Window (R-Squared)**

Note: Each plot of this figure shows the adjusted $R^2$ over time for a certain predictive horizon and the corresponding yield spread (8, 12, 16 and 20 quarters). The date on the horizontal axis determines the end date of the sample period. The first point in each graph indicates the adjusted $R^2$ for the period 1960:Q1-1995:Q4. The EFM model is reestimated at every quarter using an expanding window.
Appendix
Figure 1: Forecasting output gap, expanding window (R-squared)

Note: Each plot of this figure shows the adjusted $R^2$ over time for a certain predictive horizon using a certain yield spread. The rows of panels define the predictive horizon (1, 4, and 8 quarters) and the columns of panels the maturity of the yield spread used in the regression (4, 20, and 40 quarters). The date on the horizontal axis determines the end date of the sample period. The first point in each graph indicates the adjusted $R^2$ for the period 1960:Q1-1995:Q4. The EFM model is reestimated at every quarter using an expanding window.
Finally, the parameters refer to the following state space system:

\[ y_t = \mathbf{X} \mathbf{Z}_t + \mathbf{S}_t \eta_t, \eta_t \sim \mathcal{N}(0, I) \]
\[ \mathbf{X}_t = C + \Phi \mathbf{X}_{t-1} + \Sigma \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, I) \]

where the observable and state vectors are

\[ Z_t = [\pi_t, y_t, y_{t-1}^{(1)}, \ldots, y_t^{(40)}, y_{t-1}^{(1)}, \ldots, y_t^{(4)}, \mathbf{T}_t - y_t^{(1)}; \mathbf{E}_t = y_t^{(1)}; \mathbf{G}_t - y_t^{(1)}; \mathbf{F}_{y,t}^{(4)}, \mathbf{F}_{y,t}^{(4)}, \Delta \mathbf{y}_t^{pot} ] \]
\[ X_t = [\pi_t, y_t, y_{t-1}^{(1)}, \ldots, y_t^{(1)}, \pi_t^{(1)}, \ldots, \pi_t^{(16)}; \pi_t^{(1)}, \pi_t^{(1)}, \ldots, \pi_t^{(16)}] \]

and the parameters of the transition equation are given by:

\[ C = \begin{bmatrix} C^M \\ C^l \end{bmatrix}, \quad \Phi = \begin{bmatrix} \Phi^{MM} & \Phi^{MI} & \Phi^{M\xi} \\ \Phi^{IM} & \Phi^{II} & \Phi^{I\xi} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \Sigma^{MM} & 0 & \Sigma^{M\xi} \\ \Sigma^{IM} & \Sigma^{II} & \Sigma^{I\xi} \\ 0 & 0 & \Sigma^{\xi\xi} \end{bmatrix} \]

with

\[ \begin{pmatrix} C^M \\ C^l \end{pmatrix} = \left( I - \begin{bmatrix} \Phi^{MM} & \Phi^{MI} \\ \Phi^{IM} & \Phi^{II} \end{bmatrix} \right) \begin{pmatrix} \theta_{3 \times 1} \\ \theta_{3 \times 1} \end{pmatrix} \]

Finally, the parameters \( \Lambda_0 \) and \( \Lambda_1 \) are related to the stochastic discount factor used for pricing the government bonds:

\[ m_{t+1} = \exp(-i_t - \frac{1}{2} \Lambda_0^2 \mathbf{A}_t - \Lambda_1 \varepsilon_{t+1}) \]

with \( i_t = y_t^{(1)} \) and \( \mathbf{A}_t = \Lambda_0 + \Lambda_1 X_t \).

### Table 1: Prior Distribution of the Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distr</th>
<th>Mean</th>
<th>Stdev</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi^{MM} (1, 1) )</td>
<td>\mathcal{N}</td>
<td>0.500</td>
<td>0.300</td>
</tr>
<tr>
<td>( \Phi^{MM} (2, 1) )</td>
<td>\mathcal{N}</td>
<td>-0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^{MM} (3, 1) )</td>
<td>\mathcal{N}</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^{MM} (1, 2) )</td>
<td>\mathcal{N}</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^{MM} (2, 2) )</td>
<td>\mathcal{N}</td>
<td>0.900</td>
<td>0.300</td>
</tr>
<tr>
<td>( \Phi^{MM} (3, 2) )</td>
<td>\mathcal{N}</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^{MM} (1, 3) )</td>
<td>\mathcal{N}</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^{MM} (2, 3) )</td>
<td>\mathcal{N}</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^{MM} (3, 3) )</td>
<td>\mathcal{N}</td>
<td>0.800</td>
<td>0.300</td>
</tr>
<tr>
<td>( \Phi^M(i, j) )</td>
<td>\mathcal{N}</td>
<td>0.000</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^M(3, 3) )</td>
<td>\mathcal{N}</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^M(1, 1) )</td>
<td>\mathcal{N}</td>
<td>-0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^M(2, 1) )</td>
<td>\mathcal{N}</td>
<td>0.000</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^M(3, 1) )</td>
<td>\mathcal{N}</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^M(1, 2) )</td>
<td>\mathcal{N}</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^M(2, 2) )</td>
<td>\mathcal{N}</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^M(3, 2) )</td>
<td>\mathcal{N}</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Phi^M(i, i) )</td>
<td>\mathcal{N}</td>
<td>0.600</td>
<td>0.300</td>
</tr>
<tr>
<td>( \Phi^H(i, i) )</td>
<td>\mathcal{N}</td>
<td>0.000</td>
<td>0.150</td>
</tr>
<tr>
<td>( \Sigma^{MM}(i, i) )</td>
<td>\mathcal{I}^G</td>
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<td>0.200</td>
</tr>
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<td>( \Sigma^{MM}(2, 1) )</td>
<td>\mathcal{N}</td>
<td>-0.002</td>
<td>0.002</td>
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<tr>
<td>( \Sigma^{MM}(3, 1) )</td>
<td>\mathcal{N}</td>
<td>0.002</td>
<td>0.002</td>
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<tr>
<td>( \Sigma^{MM}(3, 2) )</td>
<td>\mathcal{N}</td>
<td>-0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>( \Sigma^{MM}(3, 3) )</td>
<td>\mathcal{N}</td>
<td>0.002</td>
<td>0.002</td>
</tr>
</tbody>
</table>

* : For the uniform distribution, we report the lower and upper bounds of the support instead of the mean and standard deviation, respectively.

**Note:** The two panels of this table report the prior density of the parameters estimated in the Extended Macro-Finance (EMF) model. \( \mathcal{N} \) stands for Normal, \( \mathcal{I}^G \) for Inverse Gamma, and \( \mathcal{U} \) for Uniform distribution.
Table 2: Prior and Posterior Distribution - Phi Matrix

<table>
<thead>
<tr>
<th>Prior Distr</th>
<th>Mean</th>
<th>Stdev</th>
<th>5 %</th>
<th>16 %</th>
<th>50 %</th>
<th>84 %</th>
<th>95 %</th>
<th>Mean</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Phi(1,1))</td>
<td>(N)</td>
<td>0.500</td>
<td>0.300</td>
<td>0.398</td>
<td>0.424</td>
<td>0.457</td>
<td>0.491</td>
<td>0.514</td>
<td>0.461</td>
</tr>
<tr>
<td>(\Phi(2,1))</td>
<td>(N)</td>
<td>-0.100</td>
<td>0.150</td>
<td>-0.064</td>
<td>-0.039</td>
<td>0.000</td>
<td>0.038</td>
<td>0.061</td>
<td>0.000</td>
</tr>
<tr>
<td>(\Phi(3,1))</td>
<td>(N)</td>
<td>0.100</td>
<td>0.150</td>
<td>0.041</td>
<td>0.066</td>
<td>0.109</td>
<td>0.155</td>
<td>0.184</td>
<td>0.112</td>
</tr>
<tr>
<td>(\Phi(4,1))</td>
<td>(N)</td>
<td>0.000</td>
<td>0.150</td>
<td>-0.014</td>
<td>0.001</td>
<td>0.025</td>
<td>0.050</td>
<td>0.067</td>
<td>0.026</td>
</tr>
<tr>
<td>(\Phi(5,1))</td>
<td>(N)</td>
<td>0.000</td>
<td>0.150</td>
<td>0.020</td>
<td>0.033</td>
<td>0.053</td>
<td>0.073</td>
<td>0.087</td>
<td>0.053</td>
</tr>
<tr>
<td>(\Phi(6,1))</td>
<td>(N)</td>
<td>-0.100</td>
<td>0.150</td>
<td>-0.160</td>
<td>-0.143</td>
<td>-0.121</td>
<td>-0.097</td>
<td>-0.083</td>
<td>-0.120</td>
</tr>
<tr>
<td>(\Phi(1,2))</td>
<td>(N)</td>
<td>0.100</td>
<td>0.150</td>
<td>-0.007</td>
<td>0.003</td>
<td>0.018</td>
<td>0.032</td>
<td>0.042</td>
<td>0.017</td>
</tr>
<tr>
<td>(\Phi(2,2))</td>
<td>(N)</td>
<td>0.900</td>
<td>0.300</td>
<td>0.876</td>
<td>0.891</td>
<td>0.913</td>
<td>0.935</td>
<td>0.949</td>
<td>0.914</td>
</tr>
<tr>
<td>(\Phi(3,2))</td>
<td>(N)</td>
<td>0.100</td>
<td>0.150</td>
<td>0.032</td>
<td>0.045</td>
<td>0.064</td>
<td>0.085</td>
<td>0.100</td>
<td>0.067</td>
</tr>
<tr>
<td>(\Phi(4,2))</td>
<td>(N)</td>
<td>0.000</td>
<td>0.150</td>
<td>-0.003</td>
<td>0.003</td>
<td>0.014</td>
<td>0.025</td>
<td>0.033</td>
<td>0.015</td>
</tr>
<tr>
<td>(\Phi(5,2))</td>
<td>(N)</td>
<td>0.000</td>
<td>0.150</td>
<td>-0.044</td>
<td>-0.036</td>
<td>-0.026</td>
<td>-0.016</td>
<td>-0.010</td>
<td>-0.026</td>
</tr>
<tr>
<td>(\Phi(6,2))</td>
<td>(N)</td>
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<td>0.150</td>
<td>-0.090</td>
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<td>-0.063</td>
<td>-0.057</td>
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</tr>
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<td>-0.096</td>
<td>-0.066</td>
<td>-0.036</td>
<td>-0.015</td>
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<td>0.868</td>
<td>0.900</td>
<td>0.924</td>
<td>0.868</td>
</tr>
<tr>
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<td>-0.010</td>
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<td>0.034</td>
<td>0.044</td>
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<td>0.150</td>
<td>0.011</td>
<td>0.021</td>
<td>0.036</td>
<td>0.052</td>
<td>0.063</td>
<td>0.036</td>
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<td>0.062</td>
<td>0.035</td>
</tr>
<tr>
<td>(\Phi(1,4))</td>
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<td>-0.223</td>
<td>-0.173</td>
<td>-0.095</td>
<td>-0.017</td>
<td>0.036</td>
<td>-0.096</td>
</tr>
<tr>
<td>(\Phi(2,4))</td>
<td>(N)</td>
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<td>0.150</td>
<td>-0.457</td>
<td>-0.392</td>
<td>-0.298</td>
<td>-0.202</td>
<td>-0.139</td>
<td>-0.300</td>
</tr>
<tr>
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<td>0.150</td>
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<td>-0.289</td>
<td>-0.190</td>
<td>-0.088</td>
<td>-0.006</td>
<td>-0.193</td>
</tr>
<tr>
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<td>0.300</td>
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<td>0.629</td>
<td>0.687</td>
<td>0.750</td>
<td>0.791</td>
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<td>-0.142</td>
<td>-0.088</td>
<td>-0.040</td>
<td>-0.008</td>
<td>-0.088</td>
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<tr>
<td>(\Phi(6,4))</td>
<td>(N)</td>
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<td>0.150</td>
<td>0.003</td>
<td>0.037</td>
<td>0.086</td>
<td>0.135</td>
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<td>0.094</td>
</tr>
<tr>
<td>(\Phi(1,5))</td>
<td>(N)</td>
<td>0.100</td>
<td>0.150</td>
<td>0.282</td>
<td>0.328</td>
<td>0.396</td>
<td>0.469</td>
<td>0.515</td>
<td>0.391</td>
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<tr>
<td>(\Phi(2,5))</td>
<td>(N)</td>
<td>-0.200</td>
<td>0.150</td>
<td>-0.172</td>
<td>-0.112</td>
<td>-0.024</td>
<td>0.075</td>
<td>0.138</td>
<td>-0.026</td>
</tr>
<tr>
<td>(\Phi(3,5))</td>
<td>(N)</td>
<td>-0.100</td>
<td>0.100</td>
<td>-0.238</td>
<td>-0.188</td>
<td>-0.115</td>
<td>-0.040</td>
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<td>-0.110</td>
</tr>
<tr>
<td>(\Phi(4,5))</td>
<td>(N)</td>
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<td>0.150</td>
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<td>-0.206</td>
<td>-0.159</td>
<td>-0.113</td>
<td>-0.083</td>
<td>-0.158</td>
</tr>
<tr>
<td>(\Phi(5,5))</td>
<td>(N)</td>
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<td>0.300</td>
<td>0.610</td>
<td>0.649</td>
<td>0.700</td>
<td>0.749</td>
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<td>0.701</td>
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<tr>
<td>(\Phi(6,5))</td>
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<td>0.150</td>
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<td>-0.163</td>
<td>-0.134</td>
<td>-0.103</td>
<td>-0.080</td>
<td>-0.134</td>
</tr>
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<td>(\Phi(6,6))</td>
<td>(N)</td>
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<td>0.719</td>
<td>0.767</td>
<td>0.808</td>
<td>0.836</td>
<td>0.762</td>
</tr>
</tbody>
</table>

Note: This table reports the prior and posterior density of \(\Phi\) in equation (7). The second to fourth columns report the type of distribution, mean and standard deviation of the prior, respectively. The fifth to ninth columns report the 5-th, 16-th, 50-th, 84-th and 95-th percentile of the posterior distribution, respectively. The last two columns report the mean and mode of the posterior distribution. The results are obtained using the Metropolis-Hastings algorithm. \(N\) stands for Normal distribution.
<table>
<thead>
<tr>
<th>Prior</th>
<th>Posterior</th>
<th>5 %</th>
<th>16 %</th>
<th>50 %</th>
<th>84 %</th>
<th>95 %</th>
<th>Mean</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma(1,1)$</td>
<td>$\mathcal{IG}$</td>
<td>0.010</td>
<td>0.200</td>
<td>1.088</td>
<td>1.125</td>
<td>1.183</td>
<td>1.247</td>
<td>1.290</td>
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<tr>
<td>$\Sigma(2,1)$</td>
<td>$\mathcal{N}$</td>
<td>-0.002</td>
<td>0.002</td>
<td>0.200</td>
<td>0.166</td>
<td>0.111</td>
<td>-0.055</td>
<td>-0.019</td>
</tr>
<tr>
<td>$\Sigma(3,1)$</td>
<td>$\mathcal{N}$</td>
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<td>0.002</td>
<td>0.003</td>
<td>0.059</td>
<td>0.142</td>
<td>0.228</td>
<td>0.286</td>
</tr>
<tr>
<td>$\Sigma(4,1)$</td>
<td>$\mathcal{N}$</td>
<td>0.000</td>
<td>0.002</td>
<td>0.071</td>
<td>0.096</td>
<td>0.132</td>
<td>0.168</td>
<td>0.193</td>
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<tr>
<td>$\Sigma(5,1)$</td>
<td>$\mathcal{N}$</td>
<td>0.000</td>
<td>0.002</td>
<td>0.024</td>
<td>0.024</td>
<td>0.089</td>
<td>0.152</td>
<td>0.199</td>
</tr>
<tr>
<td>$\Sigma(6,1)$</td>
<td>$\mathcal{N}$</td>
<td>0.002</td>
<td>0.002</td>
<td>-0.126</td>
<td>-0.103</td>
<td>-0.066</td>
<td>-0.030</td>
<td>-0.006</td>
</tr>
<tr>
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<td>$\mathcal{IG}$</td>
<td>0.010</td>
<td>0.200</td>
<td>1.208</td>
<td>1.246</td>
<td>1.308</td>
<td>1.378</td>
<td>1.427</td>
</tr>
<tr>
<td>$\Sigma(4,2)$</td>
<td>$\mathcal{N}$</td>
<td>0.000</td>
<td>0.002</td>
<td>0.323</td>
<td>0.344</td>
<td>0.377</td>
<td>0.412</td>
<td>0.436</td>
</tr>
<tr>
<td>$\Sigma(5,2)$</td>
<td>$\mathcal{N}$</td>
<td>0.000</td>
<td>0.002</td>
<td>-0.139</td>
<td>-0.116</td>
<td>-0.085</td>
<td>-0.053</td>
<td>-0.029</td>
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<tr>
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<td>0.002</td>
<td>-0.073</td>
<td>-0.035</td>
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<td>1.308</td>
<td>1.378</td>
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<tr>
<td>$\Sigma(4,3)$</td>
<td>$\mathcal{N}$</td>
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<td>0.002</td>
<td>0.323</td>
<td>0.344</td>
<td>0.377</td>
<td>0.412</td>
<td>0.436</td>
</tr>
<tr>
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<td>$\mathcal{N}$</td>
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<td>0.002</td>
<td>-0.126</td>
<td>-0.103</td>
<td>-0.066</td>
<td>-0.030</td>
<td>-0.006</td>
</tr>
<tr>
<td>$\Sigma(6,3)$</td>
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<td>0.002</td>
<td>0.002</td>
<td>-0.302</td>
<td>-0.244</td>
<td>-0.162</td>
<td>-0.077</td>
<td>-0.026</td>
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<tr>
<td>$\Sigma(4,4)$</td>
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<td>0.200</td>
<td>0.272</td>
<td>0.286</td>
<td>0.310</td>
<td>0.336</td>
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<td>$\mathcal{N}$</td>
<td>0.000</td>
<td>0.002</td>
<td>0.001</td>
<td>0.030</td>
<td>0.073</td>
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<tr>
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<td>$\mathcal{N}$</td>
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<td>0.002</td>
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<td>-0.352</td>
<td>-0.258</td>
<td>-0.175</td>
<td>-0.117</td>
</tr>
<tr>
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<td>0.200</td>
<td>0.325</td>
<td>0.342</td>
<td>0.370</td>
<td>0.401</td>
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<td>$\mathcal{N}$</td>
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<td>0.002</td>
<td>0.249</td>
<td>0.302</td>
<td>0.386</td>
<td>0.474</td>
<td>0.536</td>
</tr>
<tr>
<td>$\Sigma(6,6)$</td>
<td>$\mathcal{IG}$</td>
<td>0.030</td>
<td>0.200</td>
<td>0.603</td>
<td>0.659</td>
<td>0.754</td>
<td>0.871</td>
<td>0.957</td>
</tr>
<tr>
<td>$\Sigma(7,7)$</td>
<td>$\mathcal{IG}$</td>
<td>0.002</td>
<td>0.200</td>
<td>0.177</td>
<td>0.186</td>
<td>0.200</td>
<td>0.216</td>
<td>0.229</td>
</tr>
<tr>
<td>$\Sigma(8,8)$</td>
<td>$\mathcal{IG}$</td>
<td>0.002</td>
<td>0.200</td>
<td>0.048</td>
<td>0.052</td>
<td>0.060</td>
<td>0.069</td>
<td>0.075</td>
</tr>
</tbody>
</table>

**Note:** This table reports the prior and posterior density of $\Sigma$ in equation (7). The second to fourth columns report the type of distribution, mean and standard deviation of the prior, respectively. The fifth to ninth columns report the 5-th, 16-th, 50-th, 84-th and 95-th percentile of the posterior distribution, respectively. The last two columns report the mean and mode of the posterior distribution. The statistics of the posterior distribution are multiplied by 100. The results are obtained using the Metropolis-Hastings algorithm. $\mathcal{N}$ stands for Normal and $\mathcal{IG}$ for Inverse Gamma distribution.
**Table 4: Prior and Posterior Distribution - Remaining Parameters**

<table>
<thead>
<tr>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
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<td>Stddev</td>
</tr>
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<td>( \mathcal{N} ) -1.000 1.000</td>
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</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>-2.552</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>-0.438</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>-1.356</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>-1.633</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>0.398</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>-1.225</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>-2.448</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>-5.347</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
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</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
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<td>0.000</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>( \mathcal{N} ) -1.000 1.000</td>
<td>-2.448</td>
</tr>
</tbody>
</table>

For the uniform distribution, we report the lower and upper bounds of the support instead of the mean and standard deviation, respectively.

Note: This table reports the prior and posterior density of \( S \) in equation (23), \( \alpha_0 \) and \( \alpha_1 \) in equation (2), \( C \) in equation (2), and the initial values of the state variables, \( X_0 \). The second to fourth columns report the type of distribution, mean and standard deviation of the prior, respectively. The fifth to ninth columns report the 5-th, 16-th, 50-th, 84-th, and 95-th percentile of the posterior distribution, respectively. The last two columns report the mean and mode of the posterior distribution. All results are obtained using the Metropolis-Hastings algorithm. The statistics of the posterior distribution of \( S \) and \( A \) are multiplied by 100.

\( \mathcal{N} \) stands for Normal and \( \mathcal{U} \) for Uniform distribution.